SYNTHESIS OF NONLOCAL, NONLINEAR, AND NOISY MODELS OF SEDIMENT TRANSPORT: APPLICATION TO PLANETARY LANDFORM EVOLUTION MODELING.

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Introduction: The nature of sediment transport is fundamental to our understanding of landform evolution on planetary surfaces. The simplest physical model of sediment transport is that of linear diffusion, where the instantaneous sediment flux at some location is linearly dependent on the topographic gradient (slope) at that point [1]. Progress has been made in recent years toward understanding sediment transport beyond this simple diffusional model in three key areas: 1. *Nonlocal diffusion* allows spatial variations in sediment transport beyond those at a single point, taking into account far-field effects of topography [2-4]. 2. *Nonlinear diffusion* refines the dependence of sediment flux on topographic gradient beyond a simple linear relationship by accounting for so-called "near-failure" conditions on steep slopes [5-7]. 3. *Noisy diffusion* describes stochastic components of what is otherwise a purely deterministic process [4,8-10].

No attempt has yet been made in the literature to combine these innovations into a single, generalized model of nonlocal, nonlinear, and noisy sediment transport. Here we give mathematical descriptions of these different models as well as a synthesized version that combines them. We then describe how this synthesis may be important for landform evolution modeling, particularly on Mars and other planetary bodies.

The diffusion equation: The most basic expression for sediment transport comes from the diffusion equation, which describes the random walk behavior of particles moving in a fluid [11]:

$$\frac{\partial h}{\partial t} = K\nabla^2 h \qquad (1)$$

h is the elevation and the constant K is a material property known as diffusivity. When a Gaussian white noise term η is included, the diffusion equation is known as the Edwards-Wilkinson equation [4,10]:

$$\frac{\partial h}{\partial t} = K\nabla^2 h + \eta \qquad (2)$$

Nonlocal diffusion: The Edwards-Wilkinson equation can be generalized from the above form to take into account nonlocal topographic effects. The "nonlocality" of sediment transport can be quantified by the power z of the fractional Laplacian operator ∇^z, which in the local case is always equal to 2 [4]:

$$\frac{\partial h}{\partial t} = K\nabla^z h + \eta \qquad (3)$$

The power z increases toward 3 as transport becomes increasingly nonlocal, thus most cases will fall somewhere between these two extremes. Equation 3 is known as the linear Langevin equation and is the most general nonlocal transport law [4,10].

Nonlinear diffusion: Nonlinear transport laws make an important change to the form of Equation 1. Instead of expressing the partial derivative $\partial h/\partial t$ as being proportional to the Laplacian $\nabla^2 h$, it is instead given as the spatial divergence of the sediment flux q_m, which is then a function of the gradient $\nabla h = (\partial h/\partial x + \partial h/\partial y)$ [5-7]. These are functionally the same, the only difference being that the latter allows the elevation change to be expressed as a function of gradient:

$$\frac{\partial h}{\partial t} = \nabla \bullet q_m \qquad (4)$$

$$q_m = K\nabla h = KS \qquad (5)$$

In Equation 5, the sediment flux is linearly dependent on the gradient, which is identical to the topographic slope S. Nonlinear transport laws introduce a second term for near-failure conditions, which describes the behavior of sediment flux during slope failure events such as landslides. This nonlinear term takes two forms in the literature.

Power law: A power law is used to describe the nonlinear behavior of sediment flux as it approaches some threshold slope S_t [5,7]:

$$q_m = K_s S + K_f\left(\frac{1}{1-\left(S/S_t\right)}\right) \qquad (6)$$

Each term has its own characteristic diffusivity, K_s for creep (linear) diffusivity and K_f for near-failure (nonlinear) diffusivity, with K_f typically being a few orders of magnitude higher. While this is an improvement over the simple linear diffusion model, it has limitations as it approaches the threshold slope S_t. For slopes near S_t, the denominator approaches zero, leading to a runaway condition.

Hyperbolic tangent law: A second transport law, introduced by [6], uses a hyperbolic tangent function as a best fit to empirical measurements of nonlinear slope behavior:

$$q_m = K_S S + \frac{1}{2}K_f\left[1+\tanh\left(\frac{S-S_t}{h_x}\right)\right] \qquad (7)$$

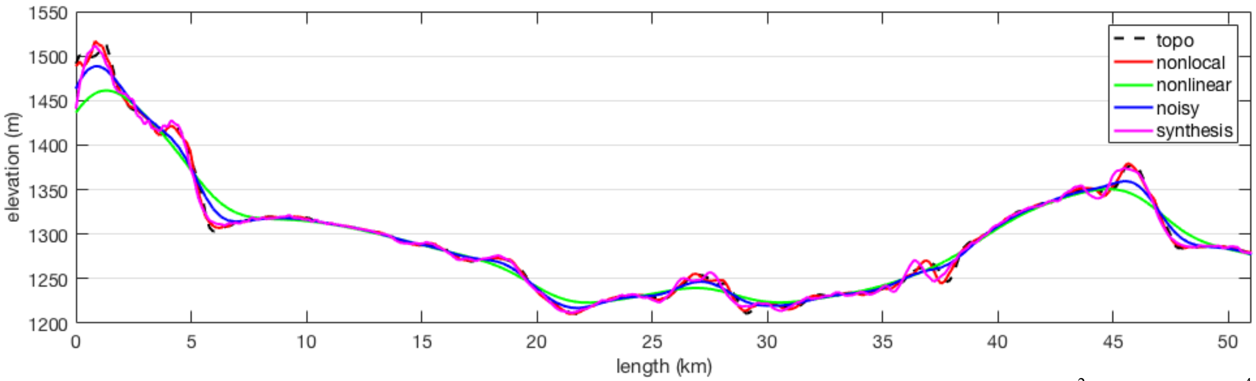

Figure 1. Simplified case of 1-D diffusional sediment transport in a crater with an integrated diffusivity ([m^2/yr]*yr) of $5x10^4$. Linear, nonlocal transport (red) better retains topographic profile but may introduce unrealistic roughness; local, nonlinear transport (green) smooths the surface to an even greater extent than local, linear transport (blue), even when Gaussian white noise is added. Synthesis model (magenta) is influenced much more by the nonlocal than the nonlinear component.

h_x is a steepness factor that determines how quickly the flux increases about the inflection point $S = S_t$. The hyperbolic tangent function is not physically meaningful in and of itself, but is more a mathematical convenience that makes a good fit to the data. Importantly, it takes into account weathering limitations on steep slopes and avoids unrealistic runaway conditions. This is the equation we will use in our synthesis model below.

Synthesis: In order to make the most physically realistic representation of sediment transport, it follows naturally to combine the above laws into a single, generalized equation:

$$\frac{\partial h}{\partial t} = \nabla^{z-1} \bullet q_m + \eta \quad (8)$$

∇^{z-1} is the nonlocal component, where z still ranges from 2-3 as in Equation 3; q_m is the nonlinear component as in Equation 7; and η is the Gaussian noise component. Such an equation can still be "localized" or "linearized" as necessary: a purely local equation would have $z = 2$; a purely linear equation would have $K_f = 0$; and a purely deterministic equation would have $\eta = 0$. Thus, Equation 8 can be used for any combination of nonlocal, nonlinear, or noisy sediment transport.

Application to planetary landform evolution modeling: Perhaps the most immediate application of the synthesis model of sediment transport is to landform evolution modeling, particularly for other planetary bodies such as Mars. For many years, the MARSSIM landform evolution model [12] has been a testing ground for simulations of fluvial flow, ponding and evaporation, groundwater seepage, and crater degradation on Mars [5,7,13-17]. The results of these simulations have been used in support of a warm and wet early Mars climate scenario, which contends that flowing liquid water was a major component of an active hydrologic system during the Noachian and Early Hesperian periods [18]. As first established by [5], MARSSIM uses a nonlinear transport law, particularly the power law form in Equation 6. It does not, however, take nonlocal topographic variations into account.

Figure 1 shows a MOLA gridded topography profile [19] for the 51-km crater Kinkora (25.0˚S, 112.9˚E) [20] with the different sediment transport regimes described here. The most significant qualitative difference is between the local and nonlocal regimes, which are controlled primarily by the power of the fractional Laplacian operator (Equation 3). Planetary applications of landform evolution modeling have not yet incorporated nonlocal sediment transport, but as can be seen here, it could be particularly important to do so in order to attain the most accurate representation of sediment transport on planetary surfaces. This could have a significant impact on the results either supporting or refuting a warm and wet early Mars climate. We explore this impact in a separate contribution [21].

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