GLOBAL LACUNARITY OF PLANETARY DATASETS: METHODOLOGY. Sheri S. Tremblay[1], Andrew J. Dombard[1], Lauren R. Schurmeier[1], and Roy E. Plotnick[1], [1]Dept. of Earth and Environmental Sciences, University of Illinois at Chicago, Chicago, IL 60607 (tremblay.sheri@gmail.com).

Introduction: Lacunarity analysis is a multi-scaled method of determining the texture associated with patterns of spatial dispersion for one-, two-, and three-dimensional data [1, 2]. In a given scale, low lacunarity geometric objects are homogeneous and translationally invariant because all gap sizes are the same whereas high lacunarity objects are heterogeneous and not translationally invariant with a wide range of gap sizes [1, 2]. Thus, lacunarity can be thought of as the 'gappiness' or 'hole-iness' in a geometric structure [1, 2].

Lacunarity, as it is simply done on a map, takes a grid and moves a box of different sizes across data by the "gliding box" algorithm [1, 2]. The gliding box algorithm successively lays sampling squares over a map so that each square uniformly overlaps its neighbors. Within each consecutive box, it will have an arbitrary value of the sample being measured (numbers, pixels, elements, etc.). The variance and mean of all the boxes are calculated and compared to determine lacunarity: $L = var/mean^2 + 1$. For evenly distributed data, each box looks like the next, the variance is low, and the resultant lacunarity value is low. For clustered data, the converse is true. This process is repeated for progressively bigger boxes to explore the clustering at different spatial scales.

Lacunarity thus requires uniform sampling of the domain, and the gliding box algorithm thus breaks down on global scales. For one, the box corners will overlap at different angles at different positions on the sphere. Additionally, the simple translations of the boxes that can be done at the small scale (simple iterations of latitude and longitude in a small scale map) will fail, particular at the poles. Thus, we have developed a methodology for looking at global datasets.

Method: Platonic solids are a collection of shapes whose vertices are uniformly distributed on a sphere. Of these, the 20-sided icosahedron places the most vertices on the sphere. We then subdivide each resultant triangular spherical plate into a triangular grid, following the algorithms in Moore [3]. There are slight variances in the resultant grid, particularly near the vertices of the icosahedron. To test the impact, we randomly rotate the grid coordinates and recalculate a lacunarity curve, finding negligible differences.

Across this grid, we move a circular window of a specified angular radius at least as big as the angular separation between the grid points. This combination of the near uniform grid and circular windows permits uniform sampling of global datasets. Like the standard gliding box algorithm, we determine the data value for each window of a given size, compare the variance and mean, then progressively increase window size towards an absolute maximum of 180°.

Application: To demonstrate the method's utility, we consider global datasets of point data (Venus's craters) and mapped data (Lunar elemental composition).

Venus's craters. Venus only possesses ~1000 impact craters, seemingly evenly distributed across its surface. Previous work using nearest-neighbor analysis found that the crater population is statistically indistinguishable from a random population, meaning the craters may or may not be clustered [4]. Nearest-neighbor analysis, however, has trouble with sparse data, while lacunarity actually works quite well with sparse data.

Thus, we examine the Venus crater databases maintained by the Lunar and Planetary Institute (https://www.lpi.usra.edu/resources/vc/vchome.html) and the US Geological Survey (https://astrogeology.usgs.gov/search/map/Venus/venuscraters). As a comparison, we generate a suite of randomly distributed points of equal numbers and a largely random population with a slight bias towards polar regions.

Lacunarity curves are shown in Fig. 1. Confirming the previous study, we find that Venus's craters are statistically indistinguishable from a random population. Indeed, this shape is typical of random populations. Lacunarity can be high at small windows sizes because some windows might fit between the random points. This possibility decreases as window size increases, dropping the lacunarity value. These curves are also coincident with that of a population with a slight polar bias. Venus's craters may be random or not [cf. 4].

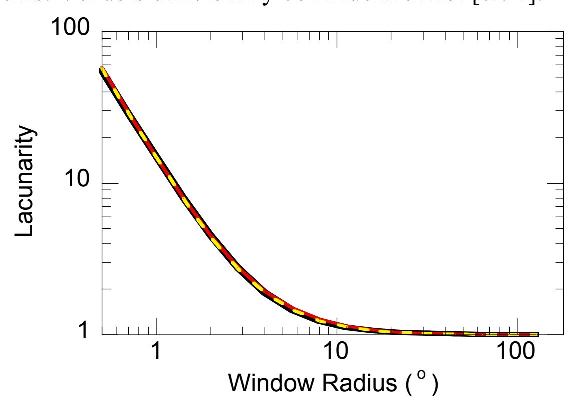

Figure 1. The lacunarity of Venus's craters (black), a random population (red), and a random population with a slight polar bias (dashed yellow).

Lunar compositional maps. Data was sourced from the Lunar Prospector Gamma Ray Spectrometer (GRS), which determined global maps of elemental abundances of uranium, thorium, potassium, and the major oxides [5]. In particular, we use the 2-degree resolution data, resampled to a uniform latitude-longitude grid. For each window in the lacunarity calculation, we find an average value of all data points that fall within the window.

Lacunarity curves are shown in Fig. 2, based on the maps shown in Fig. 3. Because of differing lacunarity values for each element, we normalize all curves to a common maximum. The lacunarity curve of CaO looks largely random, because there is only a slight bias to lower values in the Procellarum KREEP Terrane (PKT). In stark contrast, FeO, TiO_2, K, U, and Th show strong clustering, not surprising given their association with the PKT and its mare basalts. Indeed, the 30° window radius at which the lacunarity curves fall off is coincident with the roughly thousand kilometer scale of the PKT. These basalts cover the primordial Feldspathic Highlands Terrane, such that a hole is cut out of the global, largely random distribution of Al_2O_3 and SiO_2. Consequently, the lacunarity curves for these elements display characteristics of both random and clustered populations. MgO shows a similar curve; it is clustered in association of the PKT and the South Polar Aitken Basin but not nearly as strongly clustered as FeO. Consequently, the signal in the lacunarity associated with the clustering does not overwhelm the random fluctuations, and the final curve displays both characteristics.

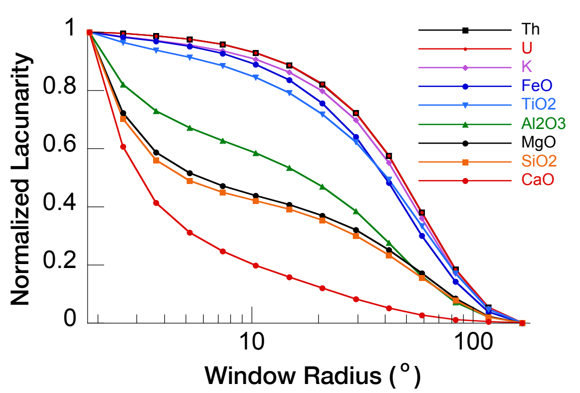

Figure 2. Log of normalized lacunarity curves for 9 different elements on the Moon.

Conclusions: We have developed a methodology using lacunarity on a sphere that can be used to explore the spatial characteristics of global datasets. Examination of Venus's crater population confirms that it is statistically indistinguishable from a random population. In addition, lacunarity curves of elemental composition maps of the Moon reveal the spatial clustering of particularly elements in the Procellarum KREEP Terrane.

Thus, lacunarity is a powerful tool for statistical analysis of global data sets. It could find utility as global planetary datasets become more common and abundant (e.g., global studies of small craters to look for those crater sizes that are contaminated clustered secondary craters). Additionally, it could be useful for other types of spherical data. For example, this methodology may be useful in astrophysics (galaxy positions, anomalies in the Cosmic Microwave Background, etc.).

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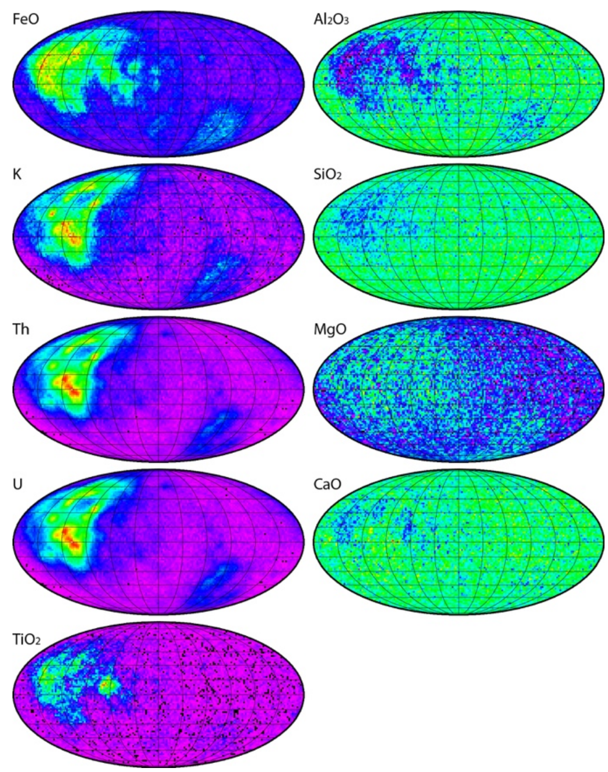

Figure 3. Global maps of Lunar elemental composition. Warmer colors are higher values.