Mass Parameter Estimation of Doubly Synchronous Binary Asteroid Systems Through Visual Observation

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Introduction: Binary asteroids represent a unique scientific target for planetary scientists both as a window into solar system formation and evolution as well as providing insight into the evolutionary dynamics of these bodies. Upcoming NASA and ESA missions expect to visit binary systems and will need accurate mass distribution models and estimation techniques to accomplish their scientific goals. This study investigates a method of estimating doubly synchronous binary asteroid system mass parameters by observation of oscillations of each body about the doubly synchronous equilibrium and the mutual orbit rate of the system. Using differential correction methods these observations allow us to recreate the observed system behavior by adjusting the mass parameters of a simulated binary model. To accomplish this, we leverage the three-dimensional arbitrary expansion order full two-body problem (F2BP) as presented in Hou 2016 [1]. A system diagram for this form of the F2BP is shown in Fig. 1, demonstrating the high dimensionality and potential complexity of the three-dimensional system.

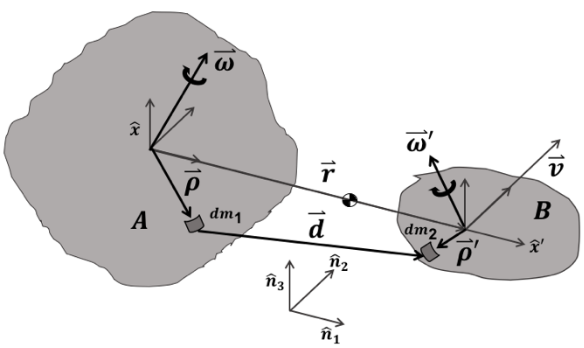

Figure 1: System diagram for the relative F2BP, where $\vec{\boldsymbol{R}}$ and $\vec{\boldsymbol{v}}$ are the relative position and velocity of the system and $\vec{\boldsymbol{\omega}}$ and $\vec{\boldsymbol{\omega}}'$ are the angular velocities of each body.

In order to evaluate the accuracy of our methodology we apply the technique to 617 Patroclus a doubly synchronous binary asteroid system and the target of a LUCY mission flyby [2].

Dynamics Model: The dynamics model implemented herein leverages inertia integrals in the mutual gravity potential to enable arbitrary order simulation of arbitrarily shaped constant density bodies. Inertia integrals, described in Eq. 1, are analogous to moments and products of inertia, but are computed for a series of expansion orders to describe more complex mass distributions, much like spherical harmonics.

$$T_B^{ijk} = \frac{1}{M_B r^{i+j+k}} \int x^i y^j z^k \, dm \quad [\mathbf{1}]$$

For the purposes of this study the expansion is truncated at the second-order; at this order the inertia integrals can be related to the moments of inertia as shown in Eq. 2-5 [3].

$$\bar{I}_{xx} = T_B^{020} + T_B^{002} \quad [\mathbf{2}]$$

$$\bar{I}_{yy} = T_B^{200} + T_B^{002} \quad [\mathbf{3}]$$

$$\bar{I}_{zz} = T_B^{200} + T_B^{020} \quad [\mathbf{4}]$$

$$\bar{I}_{xy} = -T_B^{110},\ \bar{I}_{xz} = -T_B^{101},\ \bar{I}_{yz} = -T_B^{011} \quad [\mathbf{5}]$$

As a result, the mass parameters estimated are the mass of each body and the three second-order inertia integrals of each body.

Using the inertia integral form of the mutual gravity potential, the equations of motion for the system are generated. By analyzing the behavior of the equations of motion near the doubly synchronous equilibrium we are able to identify seven periods of oscillation for the system. These periods originate from the seven degrees of freedom of the system in equilibrium: three originating from the relative attitude, three from the relative positions and an additional degree of freedom originating from nutation of the system angular momentum. By additionally including the doubly synchronous orbit period, shown for a second order system in Eq. 6, we identify eight constraints to estimate the eight mass parameters of interest [4].

$$\dot{\theta}^2 = \frac{G(M_A + M_B)}{r^3}\Big[1 + \frac{3}{2r^2}\{-2\bar{I}_{A,xx} + \bar{I}_{A,yy} + \bar{I}_{A,zz} - 2\bar{I}_{B,xx} + \bar{I}_{B,yy} + \bar{I}_{B,zz}\}\Big] \quad [\mathbf{6}]$$

Mass Parameter Estimation: The estimation of the mass parameters is accomplished by evaluating the sensitivity of the eight periodicities of the system to the mass parameters of interest, as the periodicities are a function of the masses and inertia integrals. We note that all of the moments of inertia are observable due to the coupling between the attitude and translational dynamics of the system. Thus we can invert this relationship to find the mass parameters such that the observed periods match the model periods. These parameters can be iteratively modified, correcting the parameters to make the simulated periods match the observed values at each step. The differential correction technique employed to match the observed and simulated periods requires that the sensitivities of the periods to the mass parameters be linearized about the doubly synchronous equilibrium. Using this linearization, the simulated system is iterated with each step modifying its mass parameters until the periodicities of the simulated system lie within an acceptable range of the observed values of the periodicities [5]. A mathematical description of this process is described in Eq. 7 below in which the correction to the simulated mass parameters is represented as δT and the error between the observed and simulated periods is δP. Thus $\frac{\partial P}{\partial T}$ is used to describe the sensitivity of the periodicities to the mass parameters, where the * denotes that the sensitivity has been linearized and evaluated at the doubly synchronous equilibrium. At each iteration step the correction δT is applied to the mass parameters of interest and the dynamics of the simulated system are reevaluated to update the period estimation error δP.

$$\delta T = \left[\frac{\partial P}{\partial T}\right]_*^{-1} \delta P \quad \textbf{[7]}$$

Model for Application to 617 Patroclus: Since its discovery in 2001 Patroclus has been frequently observed, however many of these observation campaigns have produced conflicting results [6][7][8]. For the purposes this investigation we choose the ellipsoidal shape models generated in Buie 2015 to generate a constant density truth model for the system. This model is selected as it provides a sufficiently complex shape for second order inertia integral analysis. In addition, the simplicity of this shape model enables simple modification of the shape for robustness analysis of the mass parameter correction method implemented for this investigation.

Observation of System Periodicities: The eight periodicities of the system are used as the observable in the estimation process. Thus an accurate method of identifying these values is necessary. Previous studies have used both stellar occultations and Hubble observations to approximate the doubly synchronous orbit rates of 617 Patroclus, however they are unable to measure the subtler motion of the system oscillations about the doubly synchronous equilibrium [6][7]. An example of the complexity of this motion is provided in Fig. 2 and 3 in which the nonlinear oscillations possible in a simplified planar system are shown. For the nonplanar problem investigated herein the nonlinear behavior will be further coupled and more complex [9].

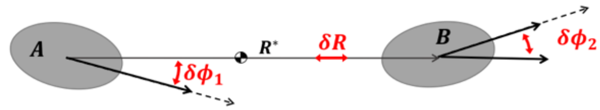

Figure 2: Two-dimensional planar diagram of oscillations showing perturbations in relative separation and two phase-angles [5].

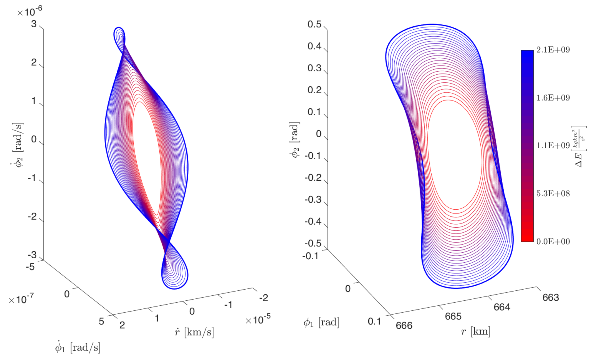

Figure 3: Example nonlinear manifold for oscillation about phase-angle ϕ_2 for the planar system showing the increase in complexity of system oscillations as energy is increased (from red to blue) [6].

For the three dimensional case the relative separation and phase angles become more intertwined as a result of the change of the relative separation from a magnitude between the body centers to a three dimensional vector. This change in the definition of the relative separation requires a redefinition of the phase angles into the relative attitude of the bodies, as the relative separation will contain some of the phase angle information in this higher dimensional system.

Potential methods for measuring these oscillations and their periods are necessary for effective implementation of our estimation technique. Thus future research will include investigations of light-curve data as a means to identify frequencies of the oscillations in addition to understanding what can be measured using occultation data and optical observations of the system as well as in-situ measurements.

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