The Systematic Structure of YORP Dynamics O. Golubov[1,2] and D.J. Scheeres[1], [1]Department of Aerospace Engineering Sciences, University of Colorado Boulder, [2]V. N. Karazin Kharkiv National University, Ukraine, scheeres@colorado.edu

Introduction: The YORP effect has been studied extensively [13], yet herein we point out a previously undiscovered symmetry that drives the obliquity and spin rate of most affected bodies in a consistent pattern, giving an unambiguous picture for how the spin rates and poles of asteroids affected by YORP will evolve. Furthermore, when combined with the Tangential YORP (TYORP) effect it enables consistent predictions of stable spin state equilibria, which are found to be possible for about 10% of the current asteroid shape models.

Background: Under the assumption of uniform rotation about their maximum moment of inertia, the secular dynamics of spin rate, ω, and obliquity, ε, are [11]:

$$I_z \frac{\mathrm{d}\omega}{\mathrm{d}t} = T_z, \qquad I_z \frac{\mathrm{d}\varepsilon}{\mathrm{d}t} = \frac{1}{\omega} T_\varepsilon. \tag{1}$$

where I_z is the asteroid's moment of inertia, t is time, and T_ω and T_ε are the axial and obliquity components of the mean YORP torque. Using the torque expressions defined in [7] and expanding these results to first order the functional form of these torques are

$$T_z = \frac{\Phi R^3}{c} C_z \left(\cos 2\varepsilon + \beta\right) \tag{2}$$

$$T_\varepsilon = \frac{\Phi R^3}{c} \alpha C_z \sin 2\epsilon \tag{3}$$

where $\alpha = 2/3$, $\beta = 1/3$, Φ is the solar energy flux at the asteroid's orbit, c is the speed of light, R is the asteroid's mean radius, and C_z is an integral over the body's surface

$$C_z = -\frac{1}{4\pi} \frac{1}{R^3} \oint_S \sin^2\psi \cos\psi \cos\eta \sin\Delta\, r \,\mathrm{d}S \tag{4}$$

The functional form of Eqns. 2 and 3 has been noted previously [14,10], although the common ratio α for both torques has not been explicitly identified and explored. Asteroids with these general functional forms for their torques were defined as Type I/II shapes, while those with additional zero crossings as a function of obliquity were defined as Type III/IV. It can be shown that inclusion of higher-order terms in our torque expansions (relevant when the leading terms are small) produce models of torques that can be classified as Type III/IV. To better represent the actual YORP effects for asteroids we can adjust the parameters α and β to match numerically computed YORP responses as a function of obliquity, using the formalism defined in [7]. Finally, to distinguish these torques (which are due only to geometry and ignore thermal inertia), we define them as the Normal YORP effect, or NYORP.

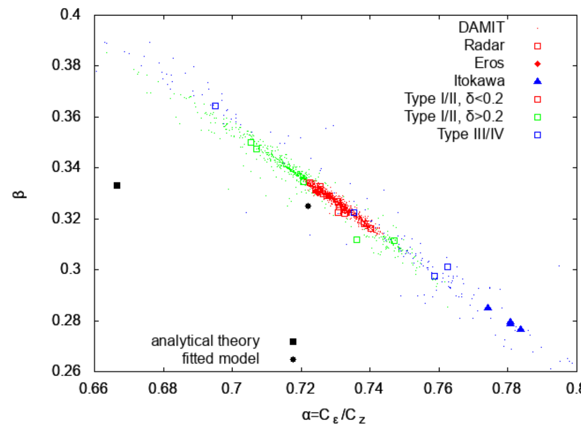

Figure 1: Correlation between the YORP coefficients $\alpha = C_z/C_\varepsilon$ and β. Different points and symbols correspond to individual asteroids. The fineness of the fit to the generic model is color-coded. The black square and circle mark theoretical predictions.

Fitted Models: To test the validity of this analytical prediction the NYORP torques were computed for several hundred shape models taking from the DAMIT database [3], radar shape models [1], and in-situ shape models of spacecraft-visited asteroids [4,5]. We find that over 80% of asteroid shapes are of Type I/II. We also apply our fitting procedure to Type III/IV shapes, to note the common trends. Figure 1 shows the best-fit values for α and β, separated by types of models and goodness of fit to the analytical form, along with the analytically predicted values. A major observation is that the factors α and β are close to the analytically derived values, and that the spin rate and obliquity torques are always of the same sign, providing a consistent migration pattern for evolving YORP spin rates.

Evolutionary Dynamics of YORP: Given this consistent pattern in NYORP torques makes it possible to lay out a generic evolutionary dynamic pathway, shown in Fig. 2 and valid when the effect of thermal inertia is ignored. Note, we choose to define the "North" pole of an asteroid to be defined by a value $C_z > 0$, and if given a shape with a negative value of C_z, can just swap its North and South poles to be consistent with this definition. Then all asteroid spin rates follow a similar pattern of increasing in magnitude as obliquity moves away from 0/180° and 90°, reach a maximum defined by the parameter β which occurs around

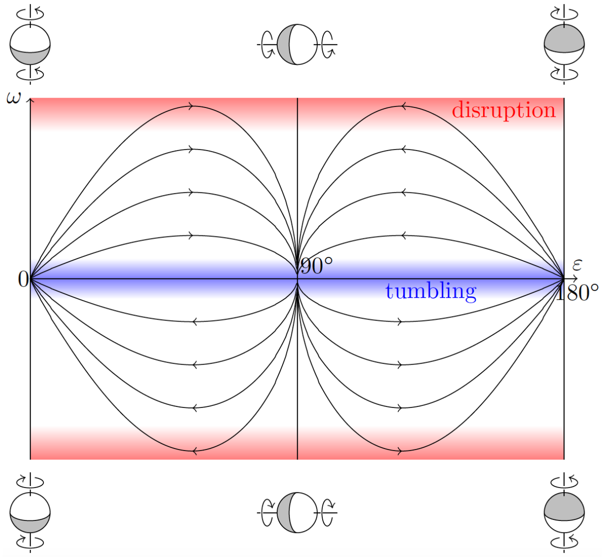

Figure 2: Generic evolutionary pathways for Type I/II asteroids subject to the Normal YORP effect.

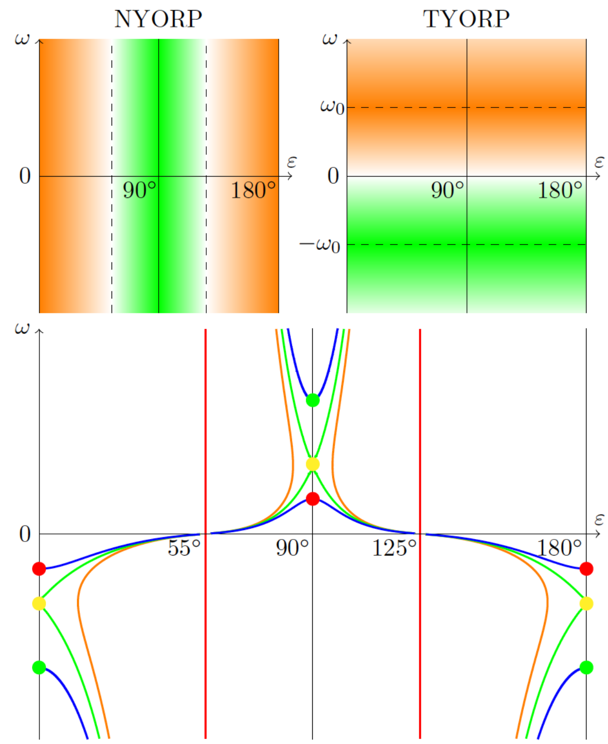

Figure 3: Generic evolutionary pathways for NYORP + TYORP dynamics. The curves outline how an increasing level of TYORP distorts the NYORP dynamics. At high enough values of TYORP spin state equilibria exist at obliquities of $0/180°$ and $90°$, denoted as green (stable) and red (unstable) dots.

$55°$, and then decrease in magnitude as they move towards obliquities of $90°$ and $0/180°$, respectively. At low spin rates the bodies should be subject to tumbling, and can emerge from this state with a different obliquity than the one with which they entered. Although such tumbling + YORP dynamics have been studied [2], this is a topic to be further understood in future research. We also note that at their maximum spin rates disruption or reconfiguration may occur, however this generically does not occur at obliquities of $0/180°$ or $90°$.

Effect of Tangential YORP: Given this systematic picture of Normal YORP dynamics, we can consider the effect of the Tangential YORP effect [8,9,12]. TYORP tends to increase the spin rate magnitude, and is due to the diurnal heating of surface boulders and rocks on asteroids. Due to its dependance on thermal inertia, it has a small effect at slow and fast spin rates, and is maximized for intermediate spin rates. Analytical models for this effect can be found in [6]. Figure 3 shows the qualitative effect of adding the Tangential YORP effect to the Normal YORP effect dynamics. The main feature of interest is the creation of equilibrium spin states where both the spin rate and obliquity remains fixed. A survey of all the asteroid shapes in the databases used show that about 10% of asteroids have values of NYORP coefficients that allow them to be trapped into such equilibria.

Conclusions: A detailed study of the YORP effect shows a significant symmetry between spin rate and obliquity dynamics that applies to all evaluated asteroid shape models. Including the effect of Tangential YORP indicates that about 10% of asteroid shapes may also have stable equilibrium states for their spin rate and obliquities. This could significantly affect the interpretation of observed asteroid spin states.

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