

THE INFLUENCE OF CURVATURE ON CONVECTION IN A TEMPERATURE-DEPENDENT VISCOSITY FLUID: IMPLICATIONS FOR THE 2D AND 3D MODELING OF MOONS. J. M. Guerrero¹, J. P. Lowman², F. Deschamps³ & P. J. Tackley⁴, ¹Department of Physics, University of Toronto, Toronto M5S 1A7, Canada (joshua.guerrero@mail.utoronto.ca), ²Department of Physical and Environmental Sciences, University of Toronto Scarborough, Toronto M1C 1A4, Canada., ³Institute of Earth Sciences, Academia Sinica, 128 Academia Road Sec. 2, Nangang, Taipei 11529, Taiwan., ⁴Department of Earth Sciences, ETH-Zurich, Sonneggstrasse 5, 8092 Zurich, Switzerland.

Introduction: Convection in the cryo-shells of icy satellites and silicate mantles of rocky moons and planets is characterized by a strongly temperature-dependent viscosity that can result in the formation of an immobile (or stagnant) conductive lid at the top of the upper thermal boundary layer of the system. Formation of a stagnant-lid occurs for heating conditions ranging from entirely internal to purely basal, and is dependent on both total energy input (i.e., internal heating rate and the rate of basal heat input) and the gradient of the viscosity variation with temperature. Consequently, the onset of stagnant-lid convection is a function of system curvature which affects the mean temperature of the convecting fluid [2,5].

Parameterizations have been obtained previously for the interior temperature and heat flux in variable core size spherical shell stagnant-lid convection systems heated solely by an isothermal core [2]. However, the authors note the absence of stagnant-lid convection when $f \leq 0.4$ in systems featuring parameter values that result in stagnant-lid convection when f is larger [2].

We extend the previous work by focussing on stagnant-lid convection in relatively small core bodies and also make comparisons of Cartesian systems to global thin shell models. To identify the parameters leading to stagnant-lid convection, we perform a large number of simulations of convection in a spherical annulus geometry [4] and a small subset featuring 3D shells. Thus, sampling the relative core-to-planet ratios relevant to Earth's Moon, the Galilean satellites, and other icy moons [e.g., 6, 7, 8, 9].

Methods: Thermally driven convection in a Boussinesq fluid with infinite Prandtl number is modeled in two-dimensional spherical annulus and three-dimensional spherical shell geometries. A hybrid finite-difference/finite-volume code, StagYY, solves the governing equations nondimensionally using a parallelized multigrid method [1, 2]. Our calculations emulate convection in a planetary mantle or ice-shell. The surface and core-mantle boundary are modeled as free-slip in each calculation and are isothermally fixed to non-dimensional temperatures of $T = 0$ and $T = 1.0$, respectively.

The controlling parameters that determine the convective regime are the ratio of core radius to outer shell radius, $f = R_c/R_p$, the reference Rayleigh number, Ra , and the Frank-Kamenetskii rheology law describing how the viscosity depends on temperature. Our reference viscosity is $\eta^* = 1.0$ at a temperature of $T = 0.5$.

Specific output variables of interest include a mid-depth Rayleigh number, Ra_m , which is obtained when the time-averaged mid-depth temperature is used to evaluate the viscosity at mid-depth and the mobility, M , which is the ratio of surface rms-velocity to interior rms-velocity. If $M \geq 0.4$ we describe the surface of the system as mobile. If $M < 0.01$ we consider the system to be in the stagnant-lid regime [3]. Intermediate values of M correspond to a transitional regime.

Results: For all models presented, we use a single Rayleigh number ($Ra = 3.2 \times 10^5$) and vary the viscosity contrast $\Delta\eta_T$ from 1 to 10^8 . We analyze stagnant-lid convection (SLC) in 2D and 3D systems with curvatures including relatively small-core shells (f as small as 0.2). Figure 1 snapshots show the effect that curvature has on the mean temperature, flow pattern and convective regime when $\Delta\eta_T$ is held fixed. Several peculiarities of convection in a strongly temperature-dependent viscosity fluid are identified for both high and low curvature systems.

We find that effective Rayleigh numbers may differ by orders of magnitude in systems with different curvatures, when all other parameters are maintained at fixed values (see Figure 2). Furthermore, as f is decreased, the nature of SLC in small core bodies shows a divergence in the temperature and velocity fields found for 2D annulus and 3D spherical shell systems (see Figure 3).

In addition, substantial differences in the behavior of thin shell ($f = 0.9$) and plane-layer (Cartesian geometry) models occur in both 2D and 3D, indicating that the latter (emulating a toroidal topology rather than spherical) may be inappropriate for modeling variable viscosity convection in thin spherical shells.

The findings are especially relevant to understanding and accurately modeling the thermal structure that may exist in bodies characterized by thin shells

(e.g., $f = 0.9$) or relatively small cores, such as shells comprising the Galilean satellites and other moons.

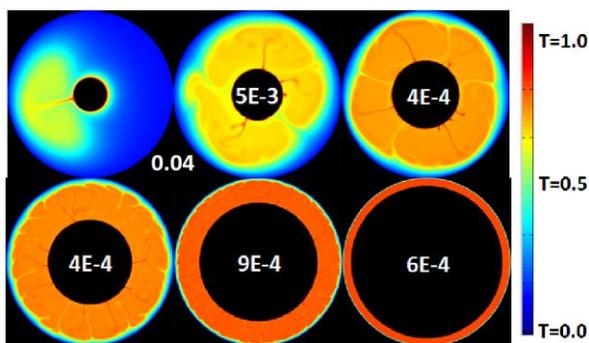


Figure 1. Selected nondimensional temperature field snapshots for spherical annulus geometry with purely basal heating. Curvature factor $f = 0.2, 0.3, 0.4, 0.5, 0.7$ and 0.9 and $\Delta\eta_T = 10^7$. Mobility values for all cases are inset. Stagnant-lid convection occurs for $f \geq 0.3$.

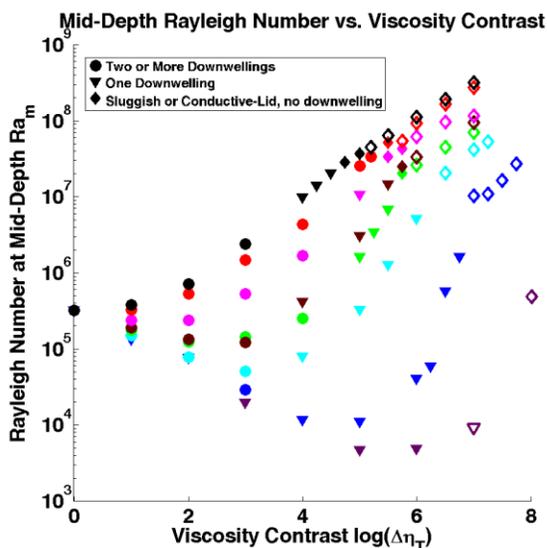


Figure 2. Mid-depth Rayleigh number as a function of viscosity contrast. Curvature is represented by the color scheme: $f = 0.2$ (purple), $f = 0.3$ (blue), $f = 0.4$ (cyan), $f = 0.5$ (green), $f = 0.55$ (brown), $f = 0.7$ (magenta), $f = 0.9$ (red), and $f = 1.0$ (black). The inset defines the correspondence between symbols and the number of downwelling observed in the case reported, where the term downwelling refers to cold fluid sourced from the upper thermal boundary layer sinking deep into the convecting system (for example, see Fig. 1). Open symbols indicate stagnant-lid convection.

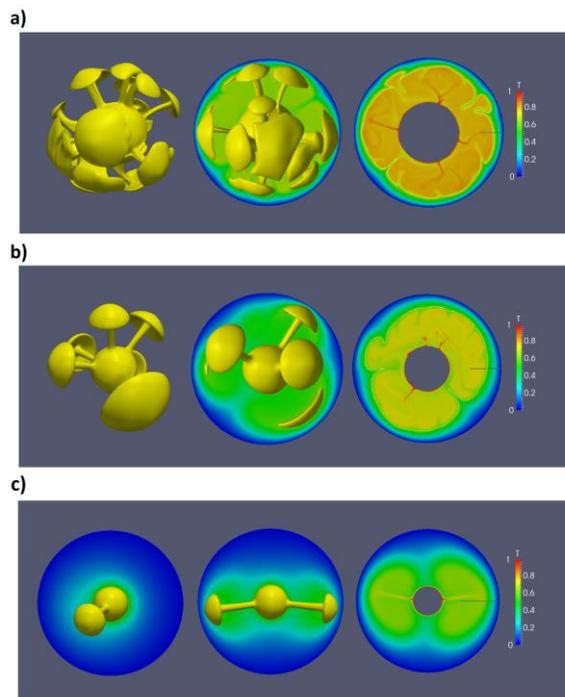


Figure 3. 2D and 3D nondimensional temperature field snapshots comparing stagnant-lid convection in small core spherical geometries. Panels show a) $f=0.4$, b) $f=0.3$, and c) $f=0.2$ spherical systems. Model parameters are identical in each panel, only the dimensionality differs. For all panels the center column shows the same isosurfaces but from a viewing angle rotated by 90 degrees in the equatorial plane, relative to the viewing angle in the left-column. The isosurfaces are superimposed on a vertical slice through the spherical shell. The right hand column shows a spherical annulus centered at the equator.

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