Determination of the Gravitational Potential and Tidal Stress of Asteroids using the Finite Element Method.

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Introduction: Because they are elementary bricks of planets during the early formation of the Solar System, asteroids have been main targets of numerous space missions such as NEAR Shoemaker [1] and Hayabusa [2], and still are with missions OSIRIS-REX [3] and Hayabusa 2 [4] arriving at their targets this year. However, little is still known about the internal structure and morphology, as only a few observables are available from Earth such as radar measurements, or gravity field measurements done by spacecraft during rendezvous missions. Here, the present work aims at determining the gravitational potentials of asteroids using the finite element method in order to compare them with the in-situ gravity field measurements but also in order to study tidal displacement and stress and thus analyse the possibility of in-situ passive seismic experiments at their surface.

Previous asteroid geodesy studies: The gravitational field of Earth is very well determined thanks to space missions like GRACE [5]. High precision models, taking into account ellipticity and local inhomogeneities have been established matching the measured gravity field, and then used to calculate deformations of the Earth [6,7]. However, these methods are well-suited for almost spherical bodies, and as such are not suited to highly non spherical asteroids with many inhomogeneities, voids and porosity [8]. These specific properties asteroids mean that there is a need to develop a specific tool to study them.

Asteroid stress and seismicity: First studies of tidal deformation of simplistic spherical asteroids [9] showed that local material failure might happen at the surface of a binary asteroid (Fig.1), and as a consequence that tidal quakes might occur, thus possibly allowing a passive seismic experiment. While global modelling of asteroids and global stress analysis have already been done [10,11], we want to study local failure. For this reason and the other mentioned above, our choice was to create a tool using the finite element method in C++/FreeFem++ [12]. The finite element method gives us freedom in discretization, mesh refining, body shapes and allows us to study local stress for failure. The theory used is the elastic deformation of a self-gravitating body, as used in Dahlen and Tromp 1998 [13]. This method solves the Poisson equation with the impulse equation and mass conservation, but without spherical symmetry meaning that it can be applied to most asteroids.

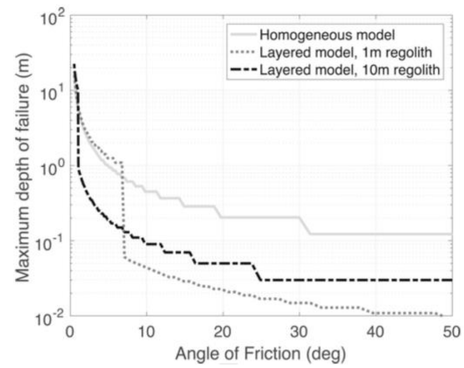
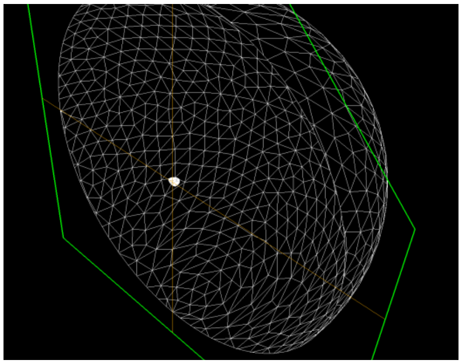


Fig.1. Depth of material failure depending on the angle of friction and internal structure of the asteroid (68503) Didymoon. The steps are due to the discretization of the models. [9]

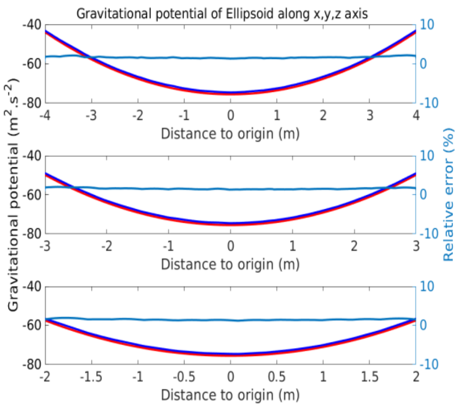
Validation: The code has been validated in two stages: first, the validation of the solution of the Poisson equation giving the gravitational field, and then the validation of the coupled system of the Poisson equation for self-gravity with the impulse equation for a deformed elastic body.

Gravitational field. Calculations of the gravitational field have been performed for a nonhomogeneous sphere and triaxial ellipsoid (Fig. 2.) and then validated via comparison with the analytical formulae from [14,15]. This allows us to apply this part of the code for the tidal displacement calculations, but also by itself for applications on asteroids whose gravity field was measured such as asteroid Eros [1,16].

Tidal displacement. Using the previous results from the gravitational equation, coupled with the impulse equation for an elastic body under a tidal potential from a main body, a moon or the Sun gives us the tidal displacement inside the studied body. Using the case of the Earth's solid tide from the Moon using the analytic formulae in a case of an incompressible homogeneous spherical Earth in [15] we were able to validate the code's calculations (Fig. 3.).



(a)



(b)

Fig.2. (a) Modelisation of a triaxial ellipsoid for calculation of its gravitational field. The large outer sphere is where the boundary condition is calculated for the Poisson equation. **(b)** Comparison of the gravitational potential of the triaxial ellipsoid with in deep blue the code result, in red the analytical value and in light blue the error between. The error stays below 1% inside the body with low computation resources due to the low mesh resolution.

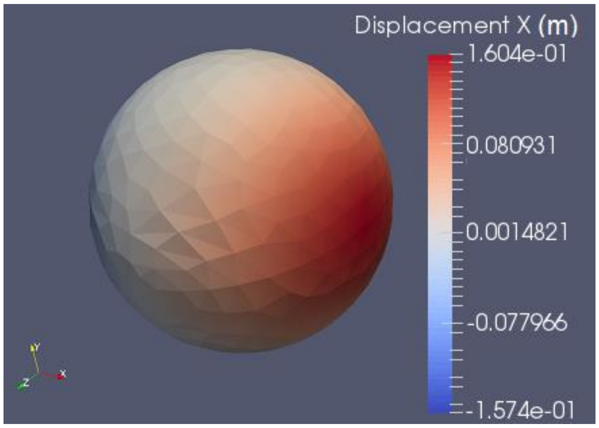


Fig.3. Tidal deformation of an elastic self-gravitating homogeneous incompressible spherical Earth under the M2 tidal potential from the Moon. Values matches analytical results from [16].

Discussion: Ongoing application of this code on the asteroid Eros for gravity field calculations and on the Martian moon Phobos for tidal deformations will be presented at the 49th LPSC conference.

References:

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