ASTEROID ORBITAL GRAVITY GRADIOMETRY. Kieran A. Carroll$^1$ and Daniel R. Faber$^2$, $^1$Gedex Systems Inc., 407 Matheson Blvd. East, Mississauga, Ontario, Canada L4Z 2H2, kieran.carroll@gedex.com, $^2$Orbit Fab, Inc., daniel@heliocentric.ca.

**Introduction:** Here we describe a novel technique for measuring the gravity field of a planetary body from orbit, and describe how that technique could be applied to determining the gravity fields of small bodies such as asteroids. In summary, the technique involves measuring the tidal acceleration field at a point offset from the mass-centre of a spacecraft, due to the gravity-gradient field of a nearby planetary body, using an absolute accelerometer.

Measuring the mass and higher-order gravity field terms from orbit has been done successfully for planets and larger asteroids and comets in the past. However, existing techniques for doing so work poorly for smaller bodies [1]. Here we focus on the case of Didymos, the ~150 m diameter secondary body of 65803 Didymos, for which the AIDA mission desires to know its mass in order to help determine the momentum transfer efficiency during the DART spacecraft impact [8]. We show the expected performance of our new technique in measuring Didymos’ mass.

**Past Techniques for Measuring Gravity From Orbit:** The simplest and earliest-used technique for measuring a body’s gravity from orbit is to measure the orbital period and semi-major axis of either a small natural moon or a small satellite orbiting about that object; Kepler’s third law then allows the primary body’s mass to be determined. This provides complete knowledge of the body’s gravity field, if the body’s mass distribution is spherically symmetric. Otherwise, the body’s gravity field may be described in terms of spherical harmonic functions; the body’s mass times G is the coefficient for the zeroth-order of these.

A variety of techniques can be used to measure orbital period and semi-major axis for a satellite orbiting a body. The earliest methods used telescopic observations from Earth, along with clocks. If the satellite carries a radio transponder, then remote observations can be made via radio tracking from Earth, measuring changes in satellite velocity over time via changes in its radio signal’s doppler shift. In that technique, doppler variations are typically processed using a kalman filter to estimate velocity and position versus time for the orbiting satellite, from which mass can be derived.

That technique has been generalized to estimate higher-order terms of the body’s gravitational potential field; also, to the case where the radio-equipped spacecraft is flying-by a body, not necessarily in orbit around it. E.g., Miller et al. [2] produced an order-10 model of 433 Eros via tracking of NEAR-Shoemaker from orbit, and Konopliv et al. [3] produced an order-20 gravity field for 4 Vesta via tracking Dawn. That has been used (e.g., for Vesta by Park et al. [4]) to constrain models of internal structure. While this technique works well for large bodies, its accuracy decreases for smaller bodies, due to the smaller body mass providing a smaller tracking signal, and due to effects from irregular body shape, as well as to uncertainty arising from solar radiation pressure (SRP) [1]. E.g., when applied to the 4 km diameter comet 67P/Churyumov-Gerasimenko [5], only an order-2 model was obtained.

Another technique suitable for smaller bodies was employed by the Hayabusa spacecraft at 25143 Itokawa (200 x 300 x 500 m in size). This involved first propulsively hovering above the asteroid, then ceasing propulsion, to fall freely, measuring height above the asteroid via LIDAR measurements. Abe et al. [9] determined Itokawa’s mass to within 5% by this means, limited by uncertainty in LIDAR measurements, and by uncertainty in the perturbing SRP force.

Another technique has two spacecraft flying in formation about a target body, measuring range-rates between them, from which gravity-field parameters are extracted (the “Low-Low Satellite-to-Satellite Tracking” technique). E.g., this has been used in the GRACE Earth gravity mission, and the GRAIL Lunar gravity mission [7]. This technique is a form of gravity gradiometry, with sensitivity proportional to the distance between the co-orbiting spacecraft. It is somewhat complex, requiring two spacecraft flying in formation, and processing the measurements involves a quite complicated nonlinear estimation process.

We have previously described a technique to measure asteroid mass for even small asteroids, by making a gravimetry measurement on its surface using an absolute gravimeter [1]. This requires a lander, which need not have a long lifetime. If the lander is also a rover, then by making measurements at multiple locations on the surface of the body, its internal mass distribution can be inferred. Gedex is developing a small (2.1 kg, <10x10x20 cm) instrument suited for this purpose, the Vector Gravimeter/Accelerometer (VEGA).

**Measuring Gravity Gradient Via Tidal Acceleration:** Near a body of mass $m$, its gravity produces a gravity gradient tensor field. If the body has a spherically symmetric mass distribution, the radial-radial (RR) component at distance $r$ from the mass centre is: 

$$V_{RR}(r) = \frac{GM}{r^3} \frac{\Delta r}{r^2}$$

where $G$ is the gravitational constant, $M$ is the mass of the body, and $\Delta r$ is the change in the distance between the probe satellite and the spacecraft. If the spacecraft is flying-by, the RR component is zero; if the spacecraft is orbiting, $\Delta r$ is the distance between the two spacecraft. If the spacecraft is so far that $r^2 \gg \Delta r$, then the RR component is proportional to $1/r^5$, and the gravity gradient field is the zeroth-order of these.

In summary, this technique combines the strengths of the gravity gradient and the orbit determination techniques, allowing mass determination to be carried out with a single spacecraft. This will be demonstrated in flight with the AIDA mission.
tensile, with magnitude \( \Gamma_{rr} = 2Gm/r^3 \); in both horizontal directions there is a compressive gradient with half that magnitude. Non-spherical shape or density variations will cause these to vary, and will add cross-component tensor terms. If one can measure a component of the gravity gradient tensor, the body’s mass can be inferred from that.

The ideal instrument for measuring this is a gravity gradiometer, but at present there are no space-flight-compatible gravity gradiometers, and all such instruments for terrestrial use are large and massive. However, one can in effect be synthesized, by mounting an accelerometer away from a spacecraft’s mass centre, to measure the tidal acceleration field, which is equal to the local gravity gradient tensor times the distance from the spacecraft’s mass centre; the tidal acceleration thus is zero at the spacecraft’s mass centre, increasing with distance from the mass centre.

This technique is most usefully carried out with an absolute accelerometer, otherwise the unknown bias in the accelerometer will contaminate the tidal acceleration measurement. The VEGA instrument is suitable for this, as it measures all 3 components of the acceleration vector with no bias. The tidal acceleration signal can be increased by mounting VEGA on a boom, to move it farther from the spacecraft’s mass centre.

**VEGA on HERA:** Here we consider the use of VEGA to measure the gravity field of Didymoon. ESA is planning a mission known as HERA (previously AIM) to rendezvous with Didymos, sometime after the impact of NASA’s DART spacecraft with Didymoon in 2022. We propose mounting a VEGA instrument on a boom to HERA, as shown with the current HERA configuration in Figure 1, in order to measure the mass of Didymoon, and potentially measure higher-order gravity field terms. This could potentially improve on mass-estimation results obtained using radio tracking of HERA from Earth, and/or optical navigation of HERA using its on-board cameras.

The gravity gradient signal is strongest close to Didymoon, so best results will obtain if HERA can approach close to Didymoon. With an estimated Didymoon mass of 3x10^9 kg, the RR component of gravity gradient will have a value of about \( 2 \times 10^{-7} \text{ m/s}^2 \) (200 E) at an altitude of 50 m above the \( \sim 150 \text{ m} \) diameter body. Approaching this close to Didymoon risks collision; we suggest that HERA could make such a close approach as part of its end-of-mission manoeuvres. This could involve making one or more slow elliptical-orbit flybys, and/or entering into a low (say, 25 m altitude) orbit about Didymoon. The latter would maximize the duration of gravity gradient signal measurement time, which will produce the most accurate mass estimate, and potentially allow determining high-order gravity field terms; from that altitude, and gravity field terms up to order 15-20 could be resolved.

Assuming VEGA is mounted on a 2.5 m deployable boom, it will be about 3.3 m from HERA’s mass centre. The RR gravity gradient component will induce a tidal acceleration of about \( 7 \times 10^{-7} \text{ m/s}^2 \) (about 70 nanoG) at VEGA’s location. VEGA’s acceleration measurement noise is expected to be close to \( 10^{-6} \text{ m/s}^2/\text{root(Hz)} \) (100 nanoG/\text{root(Hz)}); with that boom length the equivalent gravity gradiometer noise is about \( 3 \times 10^{-7} \text{ s}^{-2}/\text{root(Hz)} \) (300 E/\text{root(Hz)}). For example, consider a 5000 s low-and-slow flyby of Didymoon with a 50 m closest approach, during which an RMS accuracy of 1.4 nanoG (equivalent to 4 E RMS) is expected to be achievable. A single pass could determine Didymoon’s mass to within 2%.

Alternately, consider a case wherein HERA orbits 25 m above Didymoon’s surface, where the radial gravity gradient is \( \sim 390 \text{ E} \). Over (say) 10 days, HERA would make \( \sim 80 \) orbits, which duration would result in an RMS VEGA accuracy of about 0.1 nanoG (equivalent to 0.3 E RMS). This would allow Didymoon’s mass to be estimated to within 0.1%. The spatial resolution of the gravity gradient signal is about equal to the altitude above the surface; in this case, HERA would take about 10 minutes to travel that 25 m. A 10-minute measurement would have an RMS accuracy of about 4 nanoG (equivalent to 12 E RMS). With 80 passes, the gravity gradient due to every 25 m segment of the orbit could be determined to within 1.5 E, equivalent to the signal due to a 25 m diameter, 3.5 m thick layer with anomalous density of 100 kg/m^3.