

ON THE PROBLEM OF PLUTO’S RINGS. N. I. Perov¹, A. A. Erokhin². ¹Cultural and Educational Center named after V.V. Tereshkova, ul. Chaikovskogo, 3, Yaroslavl, 150000, Russian Federation. E-mail: perov@yarplaneta.ru. ²State Pedagogical University named after K. D. Ushinskii, ul. Respublikanskaya, 108, Yaroslavl, 150000, Russian Federation.

Introduction: In the works [1], [2] the models of dust particles motion near the planets of the Solar system have been considered in the frame of the planar circular three body problem “the Sun - a planet - a small body with negligible mass”. In the work [1] a closed trajectory of the particle in the system of Pluto-Charon is obtained by analytical way and in the work [2] closed trajectories of the particles are obtained with using of numerical integration methods. Below in the frame of the three body problem “Pluto - a satellite of Pluto - a body with negligible mass” - ring structures are localized and a condition of them existence is stated.

Based Equations: Let’s investigate in the frame of the planar three body problem motion of a particle in the gravitational fields of Pluto and Charon. It is convenient to present the differential equations of the particle motion in the forms (1) and (2) with one independent variable t which is Newtonian time [3].

$$\left(\frac{d^2x}{dt^2}\right) = -Gm_1(x-x_1)/((x-x_1)^2+y^2)^{3/2} - Gm_2(x-x_2) / ((x-x_2)^2+y^2)^{3/2} + 2(dy/dt) \cdot \omega + \omega^2 x, \tag{1}$$

$$\left(\frac{d^2y}{dt^2}\right) = -Gm_1y/((x-x_1)^2+y^2)^{3/2} - Gm_2y / ((x-x_2)^2+y^2)^{3/2} - 2(dx/dt) \cdot \omega + \omega^2 y. \tag{2}$$

Here, $m_1=1.303 \cdot 10^{22}$ kg is mass of Pluto, $m_2=1.59 \cdot 10^{21}$ kg is mass of Charon,

$$x_1 = -m_2/(m_1+m_2)$$

and

$$x_2 = m_1/(m_1+m_2)$$

are the positions of Pluto and Charon in the uniformly rotating with angular velocity ω system of coordinates with origin in the center of mass of these bodies consequently, $(y_1=y_2=0)$, x and y are coordinates of the particle in the same system of reference, G is the gravitational constant. Radii of Pluto and Charon are equal to $R_P=1187$ km and $R_{Ch}=606$ km correspondingly.

$$\omega = (G(m_1+m_2)/a^3)^{1/2},$$

where $a=19596$ km is the semi major axis of Charon.

Examples: We use the following system of units. The unit of length equals a, the unit of mass is mass of Pluto (m_1), the gravitational constant $G=1$. (The orbital period of Charon in these units equals 2π , $x_1=-0.10875$, $x_2=0.89124$). For the initial conditions equal $x_0 \neq 0$, $y_0=0$, $V_{x0}=0$, $V_{y0}=0$ (and $V_{y0} \neq 0$) the trajectories of the particles formed (any) ring structures in the Pluto’s system are presented in Fig.1–Fig.8.

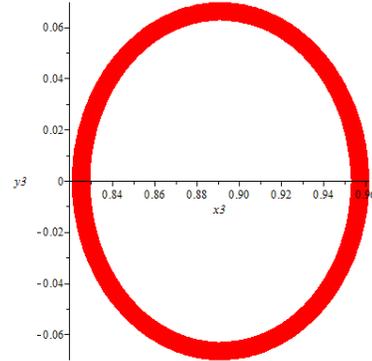


Fig.1. The (any) ring near Charon ($r_{ring}=2R_{Ch}$). $x_0=0.9532$ units of length. $V_{y0}=1.384$ (unit of length)/(unit of time). $t=100$ units of time. $N=10000$ points.

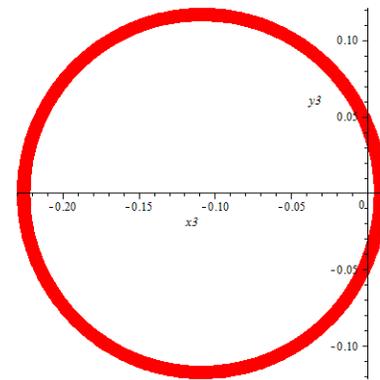


Fig.2. The (any) ring near Pluto ($r_{ring}=2R_P$). $x_0=-0.23$. $V_{y0}=-2.7$. $t=100$ units of time. $N=10000$ points.

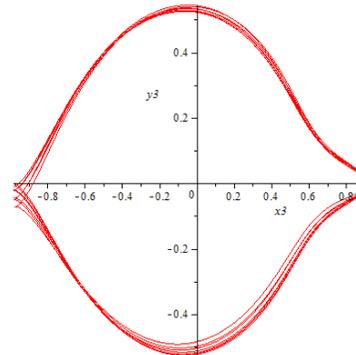


Fig.3. The (any) ring structure in the system of Pluto-Charon. $x_0=-x_2-\epsilon$. $\epsilon = 0.08964$. $V_{y0}=0$. $t=50$. $N=10000$ points.

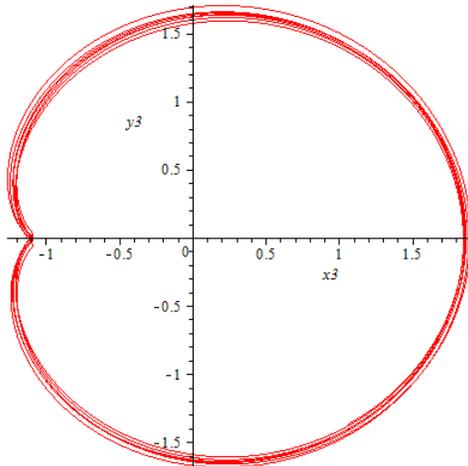


Fig.4. The (any) ring structure in the system of Pluto-Charon. $x_0=-x_2-\varepsilon$. $\varepsilon=0.197$. $V_{y0}=0$. $t=100$. $N=100000$ points.

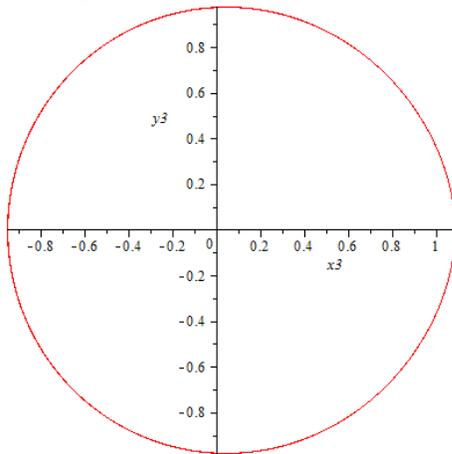


Fig.5. The (any) ring structure in the system of Pluto-Charon. $x_0=-x_2-\varepsilon$. $\varepsilon=0.06$. $V_{y0}=2.21$. $t=100$. $N=100000$ points. $t=1500$. $N=100000$ points.

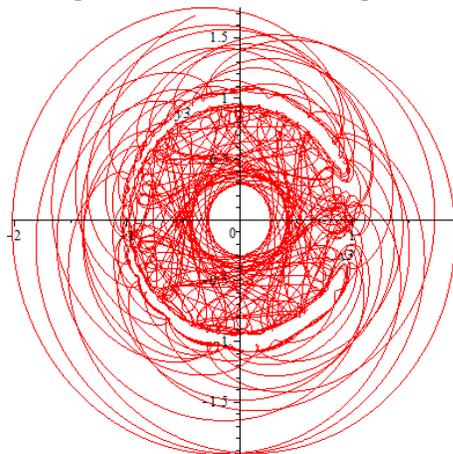


Fig.6. The chaotic trajectory of a particle in a triple system. $x_0=-x_2$. $V_{y0}=0$. $t=1500$. $N=100000$ points. $m_2'=m_2/64.5$.

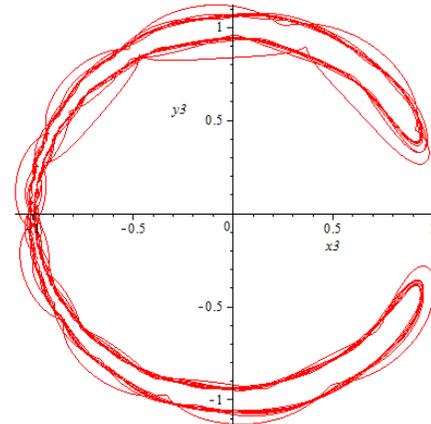


Fig.7. The transition from the chaotic trajectory to the orderly movement of a particle in a triple system. $x_0=-x_2$. $V_{y0}=0$. $t=1500$. $N=100000$ points. $m_2'=m_2/65$.

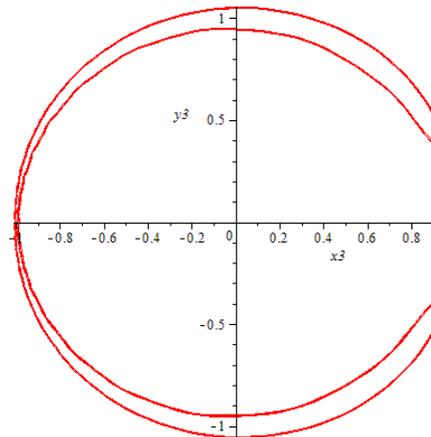


Fig.8. The orderly movement of a particle in a triple system. $x_0=-x_2$. $V_{y0}=0$. $t=1500$. $N=100000$ points. $m_2'=m_2/100$.

Conclusions: 1. For the considered model the thin and narrow (any) ring structures of the different shapes in the system of Pluto-Charon may be localized in the different parts of this system. (Fig.1- Fig.5). 2. For all the planets and their satellites of the Solar system there are the “horseshoe” trajectories if the ratio of mass (m_2') of a satellite and mass (m_1') of a planet is less than $m_2'/m_1' = 1/528.6$ (Fig.6- Fig.8). 3. There are no “horseshoe” trajectories with radius is equal to the semi major axis of Charon because $m_2/m_1=0.12 > 1/528.6$ (Fig.6). 4. The (any) “horseshoes” orbits may be found along the orbits of Styx, Nix, Kerberos and Hydra, because theirs mass is less in 528.6 times in comparison with mass of Pluto.

References: [1] Perov N.I., Sadovnikova A.A. (1994) *Astronomicheskij Vestnik*. Vol. 28. No 4-5. P. 215-222. [2] Perov N.I. (2015) *LPS XLVI*, Abstract #1021. [3] Roy A. (1978) *Orbital motion* Adam Higler Ltd. Bristol. 545 pp.