3D monostatic wavefield tomography of comet interiors

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SUMMARY

The Rosetta mission to comet 67P/C-G revolutionized comet science, but left the community strongly divided on its implications for planet formation. Are cometary nuclei "pristine" (Capria et al. 2017), implying a quiet origin and evolution, or are they "predominantly fragments resulting from collisions experienced by larger parent bodies" as predicted by the Nice model (Morbidelli and Rickman 2015)? Can these opposing views be reconciled? Although Rosetta's CONSERT radar transmission experiment at 90 MHz was unable to resolve internal structure, it proved conclusively (Kofman et al. 2015) that low frequency sounding is feasible, so that more advanced global imaging methods are practical with a dedicated radar mission.

Comet Radar Explorer (Asphaug et al. 2014) proposes to resolve this debate by acquiring a dense network of radar echoes from orbit, to obtain a high resolution 3D image that will *directly reveal the internal record of accretion and collisional evolution*. In addition to resolving the fundamental issue of planet formation, CORE will image whether cometary activity, and pervasive surface layers, are near-surface phenomena or extend to the deep interior.

Full wavefield methods (Sava et al. 2015) enable high quality 3D global imaging of internal structure of these complex, radar-transparent bodies, as characterized by contrasts and variations in dielectric (Sava & Asphaug 2018). Independent knowledge of the nucleus shape, and all-around radar acquisition from orbit, enable accurate and computationally-efficient 3D tomography.

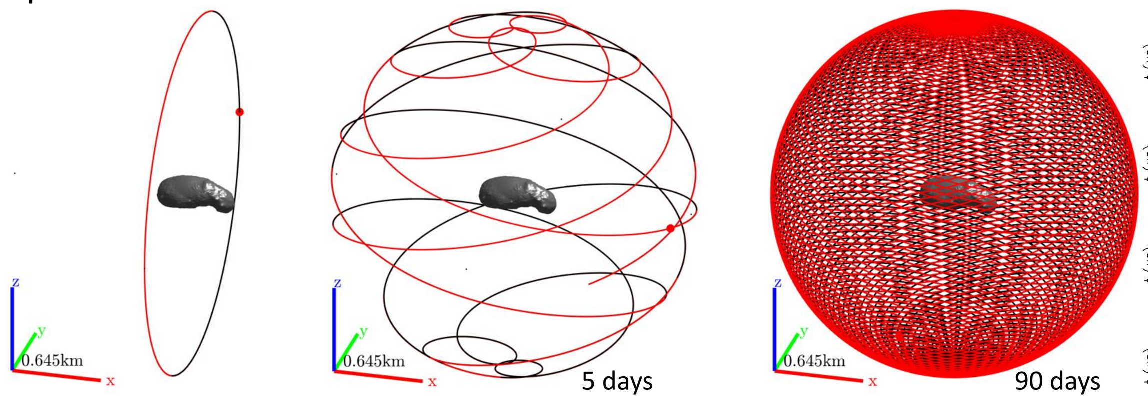
The 3D tomographic approach described here maintains similar characteristics for objects of different shapes and sizes because imaging is done in 3D and all data scattered in the interior of the comet are acquired by the orbital array. The interior resolution depends primarily on the acquisition aperture (a full sphere), frequency (as high as supported by the radar system), and sampling (several samples per wavelength obtained through sufficient orbital acquisition).

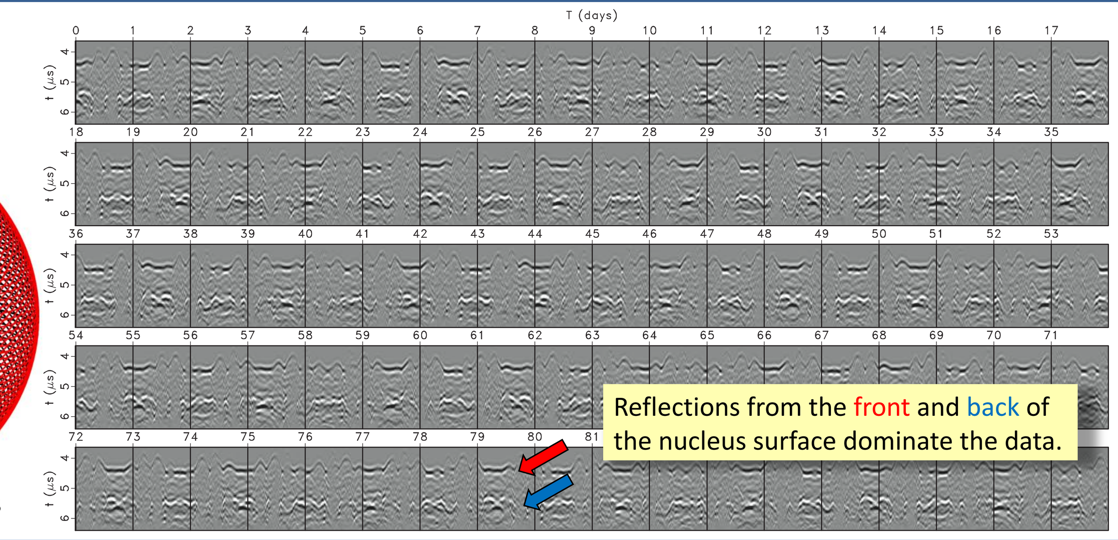
Using realistic numerical experiments, we argue that:
1.interior comet imaging is intrinsically 3D and effectively imaging the interior requires an algorithm that can take advantage of the 3D nature of the data.
2.interior tomography can be performed progressively with data acquired by successive orbits around the comet;
3.exploiting the known exterior shape of the comet facilitates cost-effective wavefield tomography;
4.multiscale wavefield tomography provides detailed information about the 3D internal physical properties;
5.monostatic tomography yields 3D models comparable with what could be accomplished by more complex systems.

REFERENCES

- Asphaug, E., et al., 2014, The Comet Radar Explorer Mission, AAS/Division for Planetary Sciences Meeting Abstracts. v. 46.
- Capria et al., 2017, How pristine is the interior of the comet 67P/Churyumov-Gerasimenko?, MNRAS, v. 469, S685–S694.
- Kofman, W., et al., 2015, Properties of the 67P/Churyumov-Gerasimenko interior revealed by CONSERT radar, Science, 349.6247, aab0639.
- Morbidelli and Rickman, 2015, Comets as collisional fragments of a primordial planetesimal disk, Astronomy and Astrophysics, v. 583, A43.
- Sava, P.C. et al., 2015, Radio Reflection Imaging of Asteroid and Comet Interiors I: Acquisition and Imaging Theory, Advances in Space Research, v. 55, 2149-2165.
- Sava, P.C., Asphaug, E., 2018, 3D radar wavefield tomography of comet interiors, Advances in Space Research, doi 10.1016/j.asr.2018.01.040.

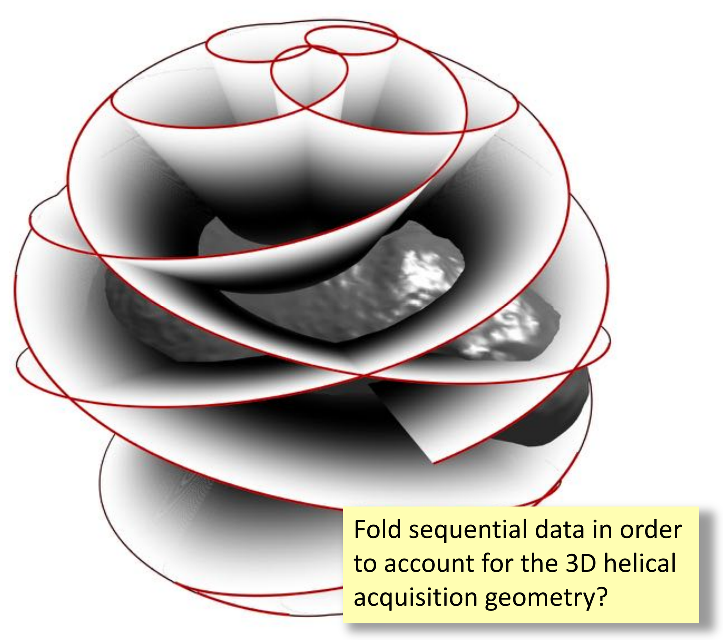
3D ORBITAL ACQUISITION





Reflections from the front and back of the nucleus surface dominate the data.

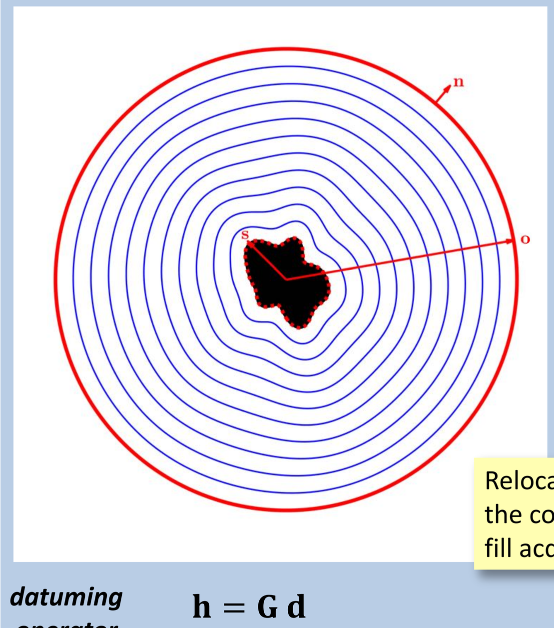
The comet nucleus imaging problem is intrinsically 3D, with all-around illumination.



Fold sequential data in order to account for the 3D helical acquisition geometry?

All reflections, and not only those originating from the nadir direction, contribute to interior imaging. Mapping 2D images in a 3D volume is difficult.

3D WAVEFIELD DATUMING



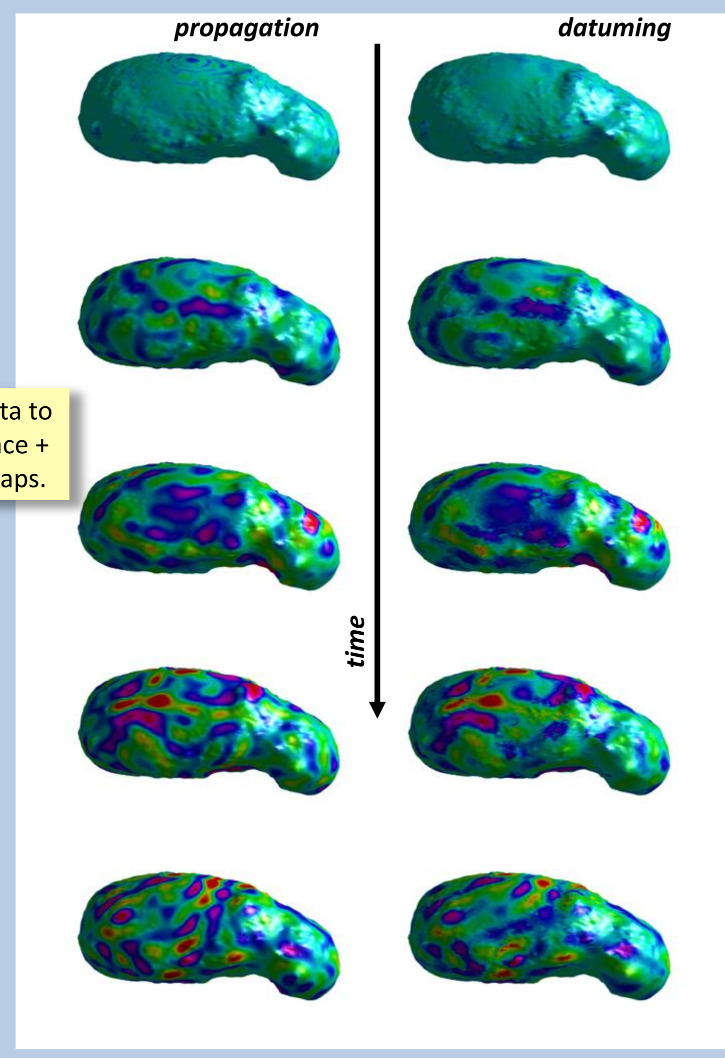
Relocate the data to the comet surface + fill acquisition gaps.

datuming operator $$\mathbf{h} = \mathbf{G}\,\mathbf{d}$$

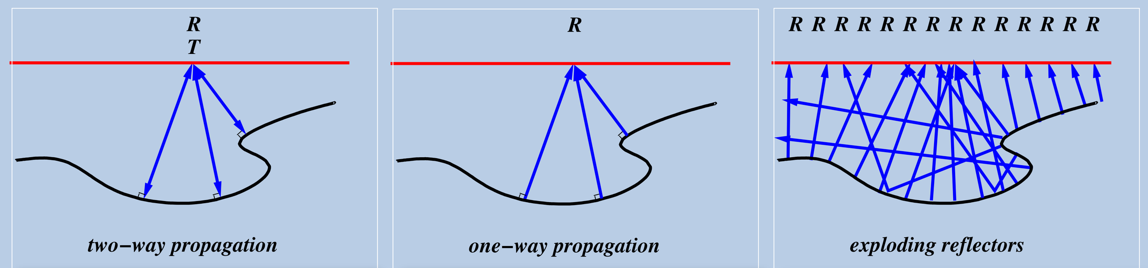
inverse problem $$J(\mathbf{d}) = \underbrace{\frac{1}{2}\|\mathbf{h} - \mathbf{G}\,\mathbf{d}\|^2}_{\textit{datuming}} + \underbrace{\frac{1}{2}\|\epsilon \mathbf{R}\mathbf{d}\|^2}_{\textit{regularization}}$$

least-squares solution $$\mathbf{d} = (\mathbf{G}^t\mathbf{G} + \epsilon^2\mathbf{R}^t\mathbf{R})^{-1}\mathbf{G}^t\mathbf{d}$$

Wavefield datuming removes propagation in empty space, and achieves computational speed-up.



EXPLODING REFLECTOR MODEL

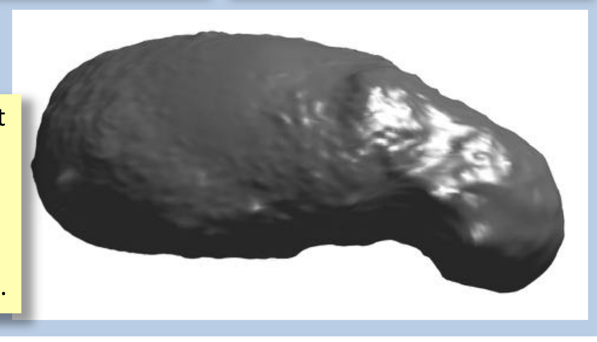


A wave propagating from transmitter *T* returns to receiver *R=T* if it reflects at 90° on an interface.

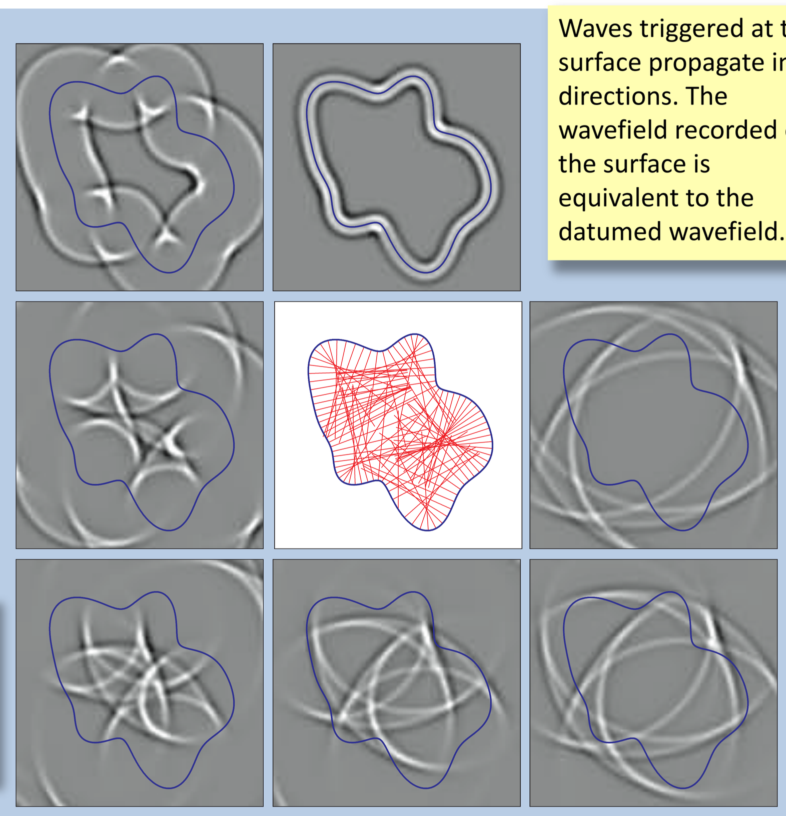
Two-way propagation can be simulated with sources on the reflector and one-way propagation.

An entire monostatic dataset can be simulated with all reflectors "exploding" at the same initial time.

The known comet exterior is "exploding reflector" for the 3D monostatic radar experiment.



The exploding reflector model transforms a monostatic reflection experiment into a transmission tomographic experiment.



Waves triggered at the surface propagate in all directions. The wavefield recorded on the surface is equivalent to the datumed wavefield.

3D WAVEFIELD TOMOGRAPHY

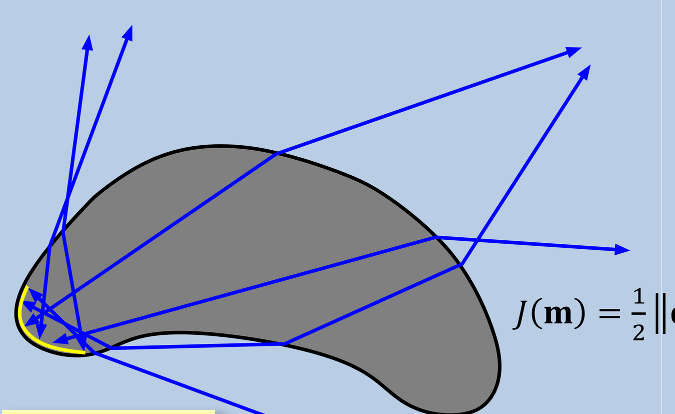


Image the back-side reflectors through the comet interior.

Formulate wavefield tomography by imposing the condition that the back-side surface reflector is imaged at its known position.

$$J(\mathbf{m}) = \frac{1}{2}\left\|\mathbf{d}^{dtm} - \mathbf{d}^{pre}(\mathbf{m})\right\|^2$$

Match wavefield simulated from the comet surface with the wavefields datumed from orbit.

$$\mathbf{d}^{pre} = \mathbf{u}(\mathbf{x} = \mathbf{s})$$

Restrict the simulated wavefeld to the surface.

Simulate wavefields using the current model.

$$m\frac{\partial^2 u(\mathbf{x},t)}{\partial t^2} - \nabla^2 u(\mathbf{x},t) = f(\mathbf{s},t)$$

Build sources from the surface reflectivity (r) and the known radar wavelet (w).

$$f(\mathbf{s},t) = r(\mathbf{s})\,w(t)$$

Iteratively update the model using non-linear optimization techniques.

$$\mathbf{m}_{k+1} = \mathbf{m}_k + \alpha_k \nabla J(\mathbf{m}_k)$$

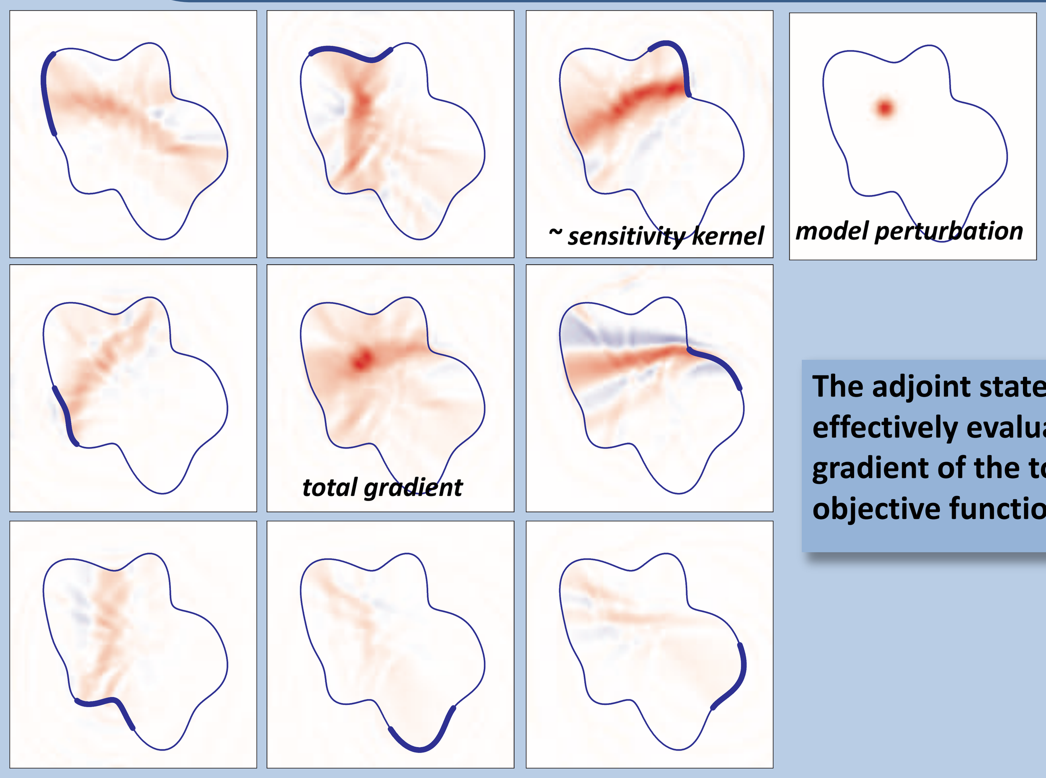
Compute the gradient efficiently using the adjoint state method.

1: compute the adjoint source $$e(\mathbf{s},t) = d^{dtm}(\mathbf{s},t) - d^{pre}(\mathbf{s},t)$$

2: simulate the state wavefield $$\mathbf{L}u \equiv m\frac{\partial^2 u}{\partial t^2} - \nabla^2 u = f(\mathbf{s},t)$$ *forward time*

3: simulate the adjoint wavefield $$\mathbf{L}^t a \equiv m\frac{\partial^2 a}{\partial t^2} - \nabla^2 a = e(\mathbf{s},t)$$ *reverse time*

4: compute the gradient $$\nabla J(\mathbf{x}) = \sum_\tau [\ddot{u}(\mathbf{x},t) \star a(\mathbf{x},t)]\delta(\tau)$$



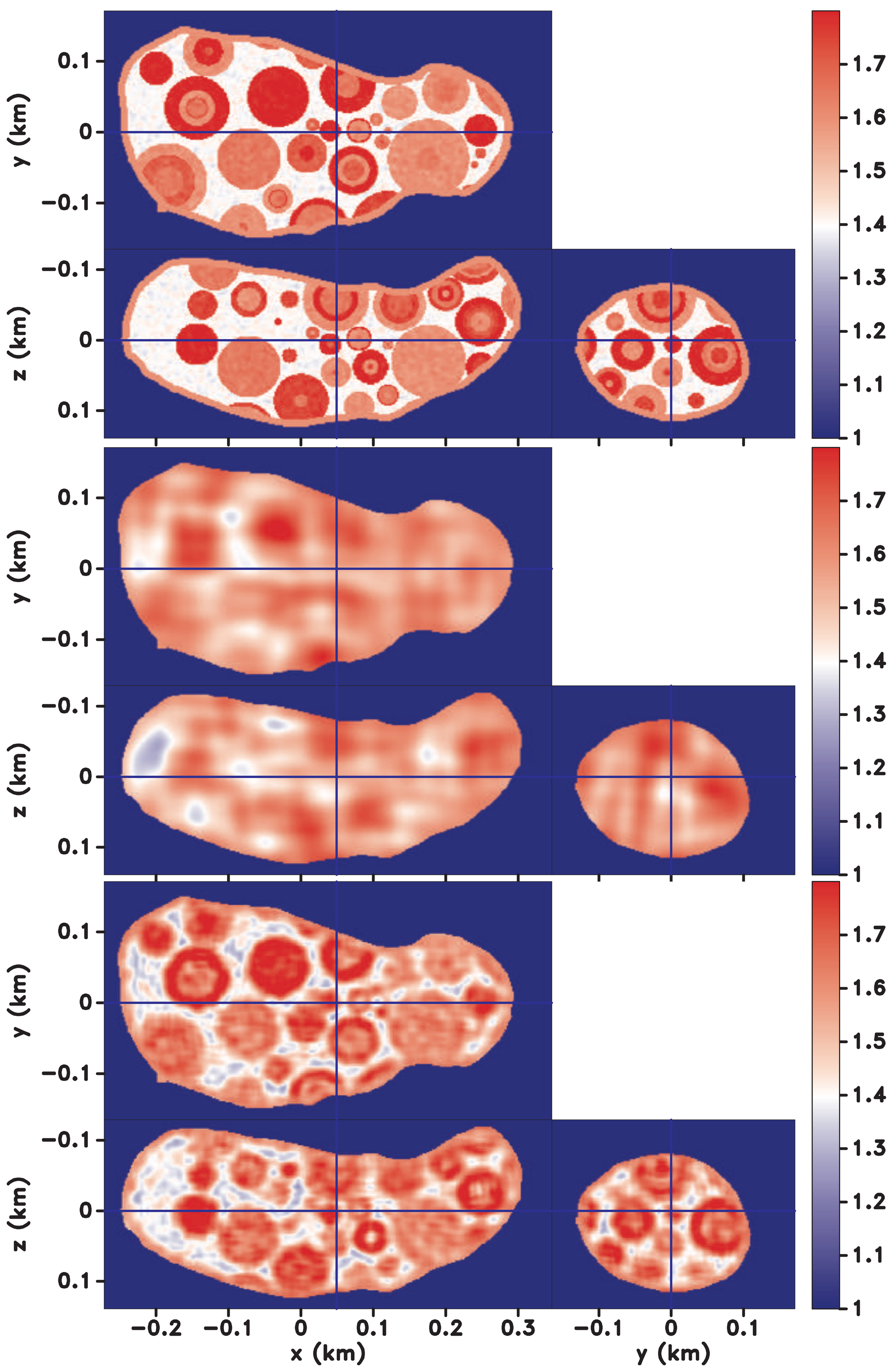
The adjoint state method effectively evaluates the gradient of the tomographic objective function.

Cometesimal pile (primordial aggregate of discrete bodies)

Implications for planet formation	What will CoRE radar see?	Indicative material properties
• Comets formed in a hierarchical accumulation of primary solid bodies. • There was limited dynamical upheaval after comet nucleus formation.	• pile of close-packed cometesimals • amorphous throughout; ices limited to outer tens of m • concentric structures inside cometesimals • compaction/deformation at cometesimal boundaries	• high contrast between cometesimals, matrix, voids, and processed surfaces • medium contrasts at compaction interfaces • low contrast structures inside cometesimals

Rubble pile (re-accumulated disruption fragments)

Implications for planet formation	What will CoRE radar see?	Indicative material properties
• Comets are collisional remnants of disrupted parent bodies. • Outer solar system underwent widespread dynamical upheaval.	• jumbled fragments of a massive disrupted parent body • size distribution of fragments • structures/layers in fragments • crystalline ices mixed throughout the comet nucleus	• high contrast at boundaries of rubble, matrix, voids and processed surfaces • low contrast structures within discrete fragments



Multi-scale wavefield tomography using monostatic reflection data recovers high-resolution models.

PROCEDURE

- Use **compact (broadband) pulses** to minimize cross-talk between reflected waves.
- Start wavefield tomography at **low frequency** (5MHz) to avoid cycle skipping.
- Progressively increase frequency to higher frequency (15MHz) to gain **model resolution**.