### Motivation

(a) The orbit of Phobos, the larger moon of Mars, is gradually spiraling inwards toward Mars
(b) With tidal stresses strengthening, in less than 70 Myr, the moon will be torn apart
(c) Development of a ring system (Black & Mittal, 2015)
(d) Ongoing satellite-ring cycle?

(Credit to Minzhi Gui)

> Did this happen in the past?

### Preprocessing the Topography Data

- Data: Mars Orbiter Laser Altimeter (MEGDRs)
- Fitting the zonal mean height from 20° N to 20° S to a diffusion equation (a hypothetical ridge)

\[
h = \frac{h_0}{2x_0} \left( \text{erf} \left( \frac{x + x_0}{\sqrt{4kt}} \right) - \text{erf} \left( \frac{x - x_0}{\sqrt{4kt}} \right) \right)
\]

- Some geological units masked out (Amazonian, early Hesperian, volcanoes and Valles Marineris) because we are searching for a >3.5 Ga ridge
- Detrending the 2D topography by subtracting a matrix derived from averaging data in radius ~2.5° range

### Searching for All Possible Paleo-equators for the Ruins of a Past Moon

- True Polar Wander may have occurred, so we took all possibilities of paleo-equators by multiplying the topography matrix by rotational matrices (parameters: rotated latitude, rotated longitude)
- For each potential topography, we used masses of initial heights \( h_0 \) and shapes \( kt \) of the ridge, and calculated the goodness of fitting between the topography and every hypothetical ridge

\[
\text{Error: } \sigma_0 = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (\text{diffusion(lat)} - \text{height}(i))^2}
\]

\[
\text{Size of ridge: } \sigma_0 = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \text{diffusion(lat)}}
\]

\[
\text{Normalized error: } r = \frac{\sigma}{\sigma_0}
\]

\[
\text{Goodness of fitting: } \omega = \sigma_0 / r
\]

Real ridge + good fitting = huge, narrow ridge and tiny error!

- Finally, we plotted goodness of fit \( \omega \) on a map. Each point on the map corresponds to a potential topography, and the value of \( \omega \) derives from the best-fitting hypothetical ridge of the topography. The higher \( \omega \), the more possibility ridge lies on the paleo-equator. (This figure shows a case with an artificially imposed ridge lying at rotated latitude ~40° & rotated longitude ~4°) - as expected, the ridge is recovered.

### Discussion

1. For Iapetus-like ridge, the more ridge lies in geological young units, the greater the limit will be, and the harder we are able to find it. For longitudinally uniform case, the ridge can never be hidden, and the upper limit value is relatively stable.
2. The degree of detrending (averaged disk size) can affect the limit. An appropriate radius should be 1° ~10°. A too big disk is ineffective for detrending, while a disk with radius less than 1° can erase the ridge even for real Iapetus.
3. At \( N = N_c = 0.2 \), the ridge ~300m in height and 6° in width. The corresponding upper limit on destroyed paleo-moon radius is <65 km, which is 6 times greater than Phobos.

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