ANGULAR MOMENTA OF COLLIDED RAREFIED PREPLANETESIMALS NEEDED FOR FORMATION OF TRANS-NEPTUNIAN SATELLITE SYSTEMS.

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Introduction: Ipatov [1] and Nesvorny et al. [2] supposed that trans-Neptunian satellite systems (e.g., binaries) were formed from rarefied condensations (rarefied preplanetesimals, RPPs). Ipatov [1,3] supposed that a considerable fraction of trans-Neptunian binaries could get the main fraction of their angular momenta due to collisions of RPPs. Nesvorny et al. [2] calculated contraction of RPPs.

Angular momentum at a collision of two preplanetesimals: The angular momentum at a collision of two RPPs (with radii r_1 and r_2 and masses m_1 and m_2) moved in circular heliocentric orbits is obtained in [1] to be equal to $K_s=k_\Theta\cdot(G\cdot M_S)^{1/2}\cdot(r_1+r_2)^2\cdot m_1\cdot m_2\cdot(m_1+m_2)^{-1}\cdot a^{-3/2}$, where G is the gravitational constant, M_S is the mass of the Sun, and the difference in semimajor axes a of RPPs equals $\Theta\cdot(r_1+r_2)$. At $r_a=(r_1+r_2)/a<<\Theta$ and $r_a<<1$, one can obtain $k_\Theta\approx(1-1.5\cdot\Theta^2)$. The mean value of $|k_\Theta|$ equals 0.6. Below we present estimates for the model for which two collided RPPs form a new RPP with mass $m=m_1+m_2$ and radius $r=(r_1^3+r_2^3)^{1/3}$. The angular velocity of such RPP equals $\omega=K_s/J_s,=2.5\cdot k_\Theta\cdot\chi^{-1}\cdot(r_1+r_2)^2\cdot r^{-2}\cdot m_1\cdot m_2\cdot(m_1+m_2)^{-2}\Omega$, where $\Omega=(G\cdot M_S)^{1/2}a^{-3/2}$ is the angular velocity of the motion of the RPP around the Sun. $J_s=0.4\cdot\chi\cdot m\cdot r^2$ is the momentum of inertia of the RPP, and $\chi=1$ for a homogeneous sphere considered in [2].

Nesvorny et al. [2] made computer simulations of compression of RPPs in the trans-Neptunian belt for initial angular velocities of RPPs equal to $\omega_o=k_\omega\Omega_o$, where $\Omega_o=(G\cdot m)^{1/2}r^{-3/2}$ is the angular velocity of the motion in a circular orbit of radius r around the gravity center of mass m. The values of k_ω were considered to be equal to 0.5, 0.75, 1 and 1.25. For most runs $r=0.6r_H$, where r_H is the Hill radius for mass m. Note that $\Omega_o/\Omega=3^{1/2}(r_H/r)^{3/2}\approx1.73(r_H/r)^{3/2}$, e.g., $\Omega_o\approx1.73\Omega$ at $r=r_H$.

At $r_1=r_2$, $r^3=2r_1^3$, $m_1=m_2=m/2$ and $\chi=1$, we have $\omega=1.25\cdot2^{1/3}k_\Theta\cdot\Omega\approx1.575k_\Theta\cdot\Omega$; e.g., $\omega\approx0.945\Omega$ at $k_\Theta=0.6$. $\omega=\omega_o$ at $r=r_H$ and $k_\omega=1.25\cdot2^{1/3}\cdot3^{-1/2}k_\Theta\cdot\chi^{-1}\approx0.909k_\Theta\cdot\chi^{-1}$, e.g. $\omega=\omega_o$ at $k_\Theta=1$, $k_\omega=0.909$, and $\chi=1$. If we consider a collision of RPPs of Hill radii r_H, and the contraction of the formed RPP to radius $r_c=0.6r_H$, then the angular velocity of the formed RPP corresponds to k_ω up to $0.909\cdot0.6^{-1/2}\approx1.17$. Nesvorny et al. [2] obtained binaries and triples only at k_ω equal to 0.5 or 0.75. Such values of k_ω can be obtained at collisions of RPPs at $k_\Theta=1$ (and $k_\omega=0.5$ can be obtained at $|k_\Theta|=0.5$). The above estimates show that initial angular velocities at which satellite systems formed in [2] could be acquired at collisions of RPPs that formed the parental RPP.

In calculations by Nesvorny et al. [2], the radius of a condensation (a RPP) exceeded $0.4r_H$. There were no calculations of compression of trans-Neptunian condensations with smaller radii. Galimov & Krivtsov [4] made calculations of compression of condensations which masses were equal to the mass of the Earth-Moon system, and which radii exceeded by a factor of 5.5 the radius of the body which mass equaled the mass of this system (i.e., the radius of the condensation was smaller by a factor of 40 than the Hill radius). For 2D runs the authors obtained formation of a binary at $0.64\le\omega_o/\Omega_o\le1.1$ (they considered $\omega_r\approx1.535\Omega_o$), and at $\omega_o/\Omega_o>1.1$ several satellites were formed. In their 3D model, satellite systems formed at $1\le\omega_o/\Omega_o\le1.5$. These values of ω_o/Ω_o exceeded by a factor of about 2 the values 0.5 и 0.75, at which formation of satellites was obtained in [2]. We can suppose that formation of satellites is possible at smaller ratios of radii of preplanetesimals to their Hill radii (for not smaller values of ω_o/Ω_o) than in [2].

Angular momentum at a collision of two preplanetesimals with initial rotation: Collided RPPs could have non-zero angular momenta before their collision. According to Safronov [5], initial angular velocity ω_{of} of a rarefied condensation was positive and equaled 0.2Ω for a spherical condensation and to 0.25Ω for a flat circle. The value of 0.2Ω is smaller than the values of ω_o considered by Nesvorny et al. [2] and is not enough for formation of satellite systems.

If two identical uniform spherical condensations (RPPs) with initial angular velocity ω_{of} collided without additional relative angular momentum, then the angular velocity of the spherical RPP that formed as a result of the collision is $\omega_2=2^{-2/3}\omega_{of}$; e.g., $\omega_2=0.126\Omega$ at $\omega_{of}=0.2\Omega$.

Let us consider the collision of two identical uniform RPPs with masses m_1 and radii $k_{col}r_H$ (where $r_H=a\cdot[m_1/(3\cdot M_S)]^{1/3}$). We consider that each RPP initially formed with radius $k_{in}r_H$ and initial angular velocity $\omega_{of}=0.2\Omega$. At such collision, the angular momentum of the formed spherical RPP is $K_s=(2k_\Theta\cdot k_{col}^2+0.16\cdot\chi\cdot k_{in}^2)\cdot r_H^2\cdot a^{-3/2}\cdot m_1\cdot G^{1/2}\cdot M_S^{1/2}\approx$ $\approx(0.96k_\Theta\cdot k_{col}^2+0.077\cdot\chi\cdot k_{in}^2)a^{1/2}\cdot m_1^{5/3}\cdot G^{1/2}\cdot M_S^{-1/6}$. In this formula at $\chi=1$, $k_\Theta=0.6$ and $k_{in}/k_{col}>2.7$ (or at

$k_{\text{in}}/k_{\text{col}}$>3.5 and $k_{\Theta}=1$) the role of initial rotation is greater than that of the collision. It shows that the role of collisions in K_s was greater only if radii of uniform RPPs differ by less than a factor of 3 from their initial radii. In the case of nonuniform (more dense to the center) RPPs, these sizes can differ by less than a factor of $3\cdot\chi^{-1/2}$.

Let us consider a collision of two RPPs with masses $k_{\text{m}}\cdot m$ and $(1-k_{\text{m}})\cdot m$ (where $0<k_{\text{m}}<1$), with initial angular velocity equal to 0.2Ω, with the same χ and initial density ρ. In this case, the component of the angular momentum K_{s} of the formed RPP caused by initial rotation equals to $K_{\text{si}}=0.2\Omega(0.4\chi\cdot m\cdot r_{in}^{2})[(1-k_{\text{m}})^{5/3}+k_{\text{m}}^{5/3}]$, where r_{in} is the radius of a RPP of mass m and initial density ρ. The component of K_{s} of the formed planetesimal r_{col} which is caused by the collision equals $K_{\text{sc}}=k_{\Theta}\cdot\Omega\cdot m\cdot r_{col}^{2}\cdot k_{\text{m}}(1-k_{\text{m}})\cdot[(1-k_{\text{m}})^{1/3}+k_{\text{m}}^{1/3}]^{2}$. If RPPs compressed before the collision than $r_{col}<r_{in}$. At $k_{\Theta}=\chi=1$ and $r_{col}=r_{in}$ the ratio $K_{\text{sc}}/K_{\text{si}}$ equals 12.5, 3 and 0.8 at k_{m} equal to 2^{-1}, 9^{-1} and 28^{-1} (i.e., at the ratio k_r of radii of collided RPPs equal to 1, 2 and 3), respectively. It means that for the considered model at the ratio k_r of radii of collided uniform preplanetesimals of different masses greater than 3, the role of initial rotation in the angular momentum K_s of the formed RPP is greater than that of the collision. If $\chi<1$, then the role of initial rotation in the angular momentum K_{s} of the formed RPP is smaller than that at $\chi=1$, and in some cases it can be smaller than that of the collision at $k_r>3$.

Angular momenta of initial RPPs were positive. Angular momenta of about 40% of observed trans-Neptunian binaries, for which inclinations of orbits of secondaries around primaries are known, are negative [6]. Such negative angular momenta could be caused by collisions of RPPs which formed parental RPPs.

Angular momentum of a preplanetesimal that grew by accumulation of small objects: If a preplanetesimal of radius $r=k_H\cdot r_H$ grew by accumulation of small objects from mass m_{o} to mass m_f at a tangential component of a velocity of a collision equal to $|v_\tau|=0.6v_c\cdot r\cdot a^{-1}$ (at $k_{\Theta}=0.6$ the same velocity was used for the above model of a collision of RPPs), then $K_{\text{s}}\approx0.173k_H^{2}G^{1/2}a^{1/2}(m_f^{5/3}-m_{\text{o}}^{5/3})M_{\text{S}}^{-1/6}\Delta K$, where $\Delta K=K^{+}-K^{-}$ is the difference between fractions of positive K^{+} and negative K^{-} increments of the angular momentum of the RPP ($K^{+}+K^{-}=1$). The values of ΔK were presented for different eccentricities of orbits and masses by Ipatov in preprints № 101 and № 102 of the Institute of Applied Mathematics of the USSR Academy of Sciences in 1981 (and in a shorter form in [7]). The obtained values of ΔK varied from -0.4 to about 1. If a radius of a RPP was about its Hill radius and objects that collided with the RPP moved before collisions in circular heliocentric orbits, then $\Delta K\approx0.9$. For more eccentric orbits the values of ΔK are typically greater. At eccentricities of heliocentric orbits $e<5(m/M_{\text{S}})^{1/3}$, ΔK exceeded 0.6 as a rule. The period of axial rotation of the preplanetesimal is $T_{\text{s}}\approx7\cdot\chi\cdot a^{3/2}\cdot(G\cdot M_{\text{S}})^{-1/2}\cdot\Delta K^{-1}$ at $|v_\tau|=0.6v_c\cdot r\cdot a^{-1}$. At $r=k_H\cdot r_H$, T_{s} does not depend on k_H. After the compression of the RPP to radius $r_c=k_{rH}\cdot r_H$ its angular velocity is $\omega_{\text{c}}\approx0.9\cdot\chi^{-1}\cdot a^{-3/2}\cdot(G\cdot M_{\text{S}})^{1/2}\cdot\Delta K\cdot k_{rH}^{-2}$. Supposing $\omega_{\text{c}}=\omega_{\text{o}}=k_\omega\Omega_{\text{o}}=k_\omega(G\cdot m)^{1/2}r_c^{-3/2}$ and $r_c\approx k_{rH}\cdot a(m/3M_{\text{S}})^{1/3}$, we can get $k_\omega\approx0.9\cdot\chi^{-1}\cdot3^{-1/2}\Delta K\cdot k_{rH}^{-2}$. At $\Delta K=0.9$ and $\chi=1$ we have $k_\omega\approx0.47k_{rH}^{-2}$, e.g., $k_\omega\approx1.3$ at $k_{rH}=0.6$, and $k_\omega\approx0.73$ at $k_{rH}=0.8$. At $k_{rH}=0.8$ and $k_H=0.75$ the radius of the formed RPP is $0.6r_H$ (as in [2]) and $k_\omega\approx0.73$.

I also considered [8] other formulas for K_{s} obtained for several other models of a growth of a preplanetesimal by accumulation of smaller objects. The estimates for the considered models showed that parental preplanetesimal with radius close to its Hill sphere could get the angular momentum at which a satellite system of a trans-Neptunian object could form. However, in this case the angular momentum of all satellite systems (e.g., binaries) would be positive. Therefore in most cases the greater fraction of the angular momentum of a preplanetesimal that contracted to form a trans-Neptunian binary was acquired at a collision of preplanetesimals, but not by accumulation of small objects. However, some fraction of the angular momentum of parental preplanetesimals could be delivered by small objects.

Conclusions: Trans-Neptunian satellite systems could form as a result of contraction of parental rarefied condensations that got the angular momenta needed for such formation at collisions of condensations.

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