Introduction: The Chelyabinsk event on 15 February 2013 brought light to the potential damage that can be caused by an airburst from a relatively small impactor [1]. Near-Earth Objects of comparable size (1-100 m diameter) maybe more abundant than previously thought [2], are difficult to observe and may strike Earth with little to no warning. This makes being able to model their potential damage quickly and accurately imperative. Several semi-analytical models have been developed to predict the atmospheric energy deposition during catastrophic fragmentation of stony meteoroids [3-5]. However, each model requires several coefficients that are difficult to constrain. Here we compare three such catastrophic fragmentation models, to determine which is the most appropriate for probabilistic hazard assessment. A more appropriate class of models exists for modelling iron meteoroids [6], which are not considered here.

Catastrophic Fragmentation Airburst Models: Three catastrophic fragmentation models are described in the literature:

- Pancake Model, Chyba et al., 1993 [3]. The simplest of the models, treats the meteoroid as a strengthless liquid that deforms due to the differential pressure across the object, promoting spreading perpendicular to the trajectory. Deformation begins when the ram pressure, \( P_{\text{ram}} = \rho_a v^2 \) (where \( v \) is impactor speed and \( \rho_a \) is local air density), exceeds the characteristic strength of the meteoroid \( \sigma_0 \), which is assumed to be constant. The extent of spreading can be controlled by a limiting factor, called the pancake factor \( f_p \), or left unconstrained. The spreading rate is affected by the impactor’s size, velocity and density, as well as the atmospheric density and drag coefficient.

- Debris Cloud Model, Hills & Goda, 1993 [4]. This model treats the fragmented meteoroid as an impermeable debris cloud that expands until it reaches a critical radius. A fragmentation cascade occurs whilst the ram pressure exceeds the strength of the material. When the cloud has slowed such that the ram pressure drops below the fragment strength, expansion stops and the fragments can be treated independently. The expansion rate, derived from an energy balance, depends on impactor’s velocity and density, the atmospheric density, and a dimensionality coefficient \( \alpha \) of order 1. It is independent of the impactor size.

- Chain Reaction Model, Avramenko et al., 2014 [5]. This model is based on an analogy of the fission chain reaction in fissile materials. Fragmentation begins when \( P_{\text{ram}} \) exceeds the effective strength of the fragments \( \sigma_e \), but in this case the fragment strength is assumed to increases as fragment size decreases, where strength follows an inverse power-law of mass with exponent \( \beta \) [7]. In contrast to the two previous models, where ablation reduces mass, but not the cross-sectional area of the deforming impactor, the radius of the expanding cloud is reduced by ablation.

Model Setup: To compare the differences between these fragmentation models, all three models were implemented using a common numerical framework to calculate the change in meteoroid velocity \( v \), mass \( m \), trajectory \( \theta \), and altitude \( z \). The Chelyabinsk airburst was well documented and provides useful calibration data. Initial velocity and trajectory were measured from observations [5], \((v_0 = 19.04 \, \text{km/s}^2, \theta = 17^\circ)\), whereas initial radius and material density are best estimates from within a range of uncertainty \([1, 5, 6]\), \( r_0 = 9.95 \, \text{m} \), \( \rho_m = 3.3 \, \text{g/cm}^3 \) respectively. For all models the same drag coefficient, \( C_D = 1.5 \), heat coefficient, \( C_H = 0.12 \) and heat of ablation, \( Q = 1 \times 10^7 \, \text{J/kg} \), were used. For each model, the various coefficients and \( \sigma_0 \) were adjusted to optimise each model’s fit to the recorded light curve energy deposition data [2]. Energy deposited in the atmosphere was calculated minus the observed radiated energy for all three models [5]. All models used the U.S. Standard Atmospheric Model for air density.

The optimised values were then used to model the Tunguska airburst, for which estimates of burst altitude and energy exist, but no light curve data was recorded. The total energy released to cause the extensive damage observed is in the range of 3-20 Mt of TNT equivalent (1 Mt TNT = 4.184 \times 10^{15} \, \text{J}) , while burst altitude estimates range from 5-10 km, with the most commonly estimated value of 8.5 km [3, 6, 8, 9]. For Tunguska, nominal impact parameters were used to produce a total energy release of 14.5 Mt; \( v_0 = 20 \, \text{km/s}^2 \), \( \theta = 45^\circ \), \( r_0 = 28 \, \text{m} \), \( \rho_m = 3.3 \, \text{g/cm}^3 \).

Results and Discussion: For Chelyabinsk, all three models can provide a good fit to the inferred energy deposition curve (Fig. 1), although each requires a different meteoroid strength. The pancake model was optimised using \( f_p = 7.3 \) and \( \sigma_0 = 0.75 \, \text{MPa} \). The debris cloud uses \( \alpha = 0.3 \) and \( \sigma_0 = 0.9 \, \text{MPa} \). The chain reaction model uses \( \beta = 0.172 \) and \( \sigma_0 = 0.45 \, \text{MPa} \). We note that these are non-unique solutions. Equivalently good fits can be obtained with other parameter combinations; however, in no case could we achieve equally good fits with identical meteoroid strengths.
The initial meteoroid strength controls the altitude of the initiation of fragmentation in each model. This together with the assumed expansion rate (Fig. 2) determines the upper portion of the energy deposition curve and the altitude of peak energy release (burst altitude). The chain reaction model requires the lowest initial strength, owing to the increase in strength as fragmentation progresses and its initially low expansion rate. The debris cloud model requires the largest initial strength, because the initial expansion rate is most rapid in this model (Fig. 2). Meteoroid expansion ends abruptly in both the pancake and debris cloud models. In contrast, in the chain reaction model meteoroid expansion eventually gives way to contraction when ablation begins to dominate and fragmentation ceases.

The lower portion of the energy deposition curve, after peak release, is set by maximum size of the fragmented meteoroid. In this case, the debris cloud model provides the best match to the observation, because in this model the meteoroid has the greatest drag and continues to decelerate most rapidly. We note that none of the models reproduce the small second peak in the data at ~24 km. This can be accounted for by a secondary strong fragment which separates from the main bolide [5] and causes the observed double smoke trail [10], but was not modelled here.

Figure 3 shows the energy deposition profiles for a Tunguska-scale airburst using the same parameters, including meteoroid strength, as employed in the Chelyabinsk calibration. For airbursts of this magnitude, meteoroid strength has less effect on the energy deposition curve so the precise assumed strength is less important. Nevertheless, all three models vary considerably in the profiles they produce. The pancake model predicts the lowest burst altitude, closest to the assumed range. The chain reaction model releases its energy over the shortest altitude range, resulting in the highest peak energy deposition, whereas the other two models release it over a more prolonged altitude range.

**Conclusions:** Each model can be tuned to fit the Chelyabinsk light curve; however, when up-scaled to a different energy each model predicts a different outcome. This inter-model uncertainty should be accounted for in probabilistic hazard assessment. Ablation and increasing strength as fragmentation progresses provide a physical rational for maximum spreading size.


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