Formation of the lunar fossil bulge and its implication for the dynamics of the early Earth and Moon. Chuan Qin[1], Shijie Zhong[1] and Roger Phillips[2], [1]Dept. of Physics, Univ. of Colorado at Boulder, Boulder, CO 80309, USA. [2]Dept. of Earth and Planet. Sci., Washington Univ. in St. Louis, MO, 63130, USA (szhong@colorado.edu).

Introduction: The gravitational and shape anomalies associated with lunar rotational and tidal bulges (i.e., l=2 features) are about one order magnitude larger than predicted from the hydrostatic theory for the Moon's present-day rotational and orbital states [1, 2]. A common hypothesis for the Moon's excess bulge is that it is a remnant feature, "frozen-in" from an early Moon that had a larger bulge because the Moon was closer to the Earth, spun faster and experienced larger rotational and tidal forces [3, 4]. As the Moon receded from the Earth, it cooled and solidified from an early magma ocean, and the early bulge may be kept or only partially relaxed [4]. This is similar to what was proposed to explain the Earth's degree-2 gravity anomalies back in 1960's [5] but has been subsequently disregarded [6].

However, the question when and how the Moon acquires the fossil bulge is largely unaddressed. A number of studies suggested that the fossil bulge was formed when the Moon was on an orbit with a semi-major axis ranging from several to 23 times of the Earth's radius, R_e [2, 4, 7]. However, these studies did not consider the physical process for forming the fossil bulge. For the Earth's fossil bulge hypothesis, a large lower mantle viscosity was invoked to maintain a fossil bulge, and the required viscosity is orders of magnitude larger than constrained by post-glacial rebound and mantle convection studies [6]. For the Moon, the surface lithospheric shell with its membrane and bending strength may be capable of maintaining a fossil bulge, as indicated in a viscoelastic deformation model [8].

To investigate the development of a lunar fossil bulge, a deformation model needs to consider viscoelastic rheology with time-dependent rotational forces due to de-spinning and lithospheric thickening due to the cooling of the Moon. The modeling presents a challenge to conventional computational techniques that are mostly applicable to problems with either time-invariant viscoelastic property [e.g., 8] or relatively short time-scales [9]. In this study, by employing a semi-analytical computational method, we illustrate the process with which a lunar fossil bulge develops with time and determine how the size of the fossil bulge depends on the Moon's de-spinning and cooling histories. Since the lunar de-spinning history is largely controlled by lunar orbital evolution and hence mostly tidal dissipation of the Earth (i.e., its Q value), our study may provide constraints on the dynamics of the early Earth and Moon (i.e., the Moon's orbital evolution and Earth's Q), by using the observation of the lunar fossil bulge.

Physical Model and Methods: The deformation at the planetary surface and interiors in our model is governed by conservation equations of mass and momentum that are coupled together with the Poisson equation for gravitational potential anomalies produced by the deformation. The deformation is driven by rotational force (the tidal force is ignored here). The lunar mantle and lithosphere are modelled as an incompressible viscoelastic, Maxwellian medium. Our model includes a core with a radius of 340 km [10].

Both de-spinning and lithospheric thickening with time are incorporated in the model. The viscosity is assumed to be temperature-dependent and evolves with time to reflect the lithospheric thickening as the Moon cools. The mantle and lithospheric temperature are determined by a 1-dimensional (i.e., in the radial direction) time-dependent heat conduction equation with an initially uniform temperature that represents the end of the magma ocean solidification. The temperature is converted into viscosity, using an activation energy of 300 KJ/mol and 10^{22} Pas viscosity for the lunar mantle interior. High viscosity occurs in the low temperature and surface regions that define the lithosphere.

Lunar spin rate is directly related to the semi-major axis of the lunar orbit for a tidally locking synchronous rotation, via the Kepler's 3rd law. Due to tidal friction that transfers angular momentum and energy between them, the Moon sees its semi-major axis increase and its spin rate decrease with time. It is possible to formulate a model for lunar de-spinning or orbital (i.e., semi-major axis) history with the Earth's Q-value as a controlling parameter [11], and a larger Q leads to a more gradual increase (decrease) in semi-major axis (spin rate) at early times in the process. In this study, we take another approach by specifying semi-major axis, a, to follow the equation: $(a-a_0)/(a_p-a_0)=(t/t_p)^b$, where a_p and a_0 are the semi-major axes at present-day time t_p and initial time t=0 (i.e., when the lunar magma ocean is solidified and a lithosphere starts to form), and b is a free parameter between 0 and 1 controlling the rate of change in semi-major axis a. a_p=60R_e and t_p is ~4 Ga, depending on when the lunar magma ocean is solidified [12]. Parameter b has a similar physical meaning to Q in that a smaller b leads to more rapid increase (decrease) in a (spin rate Ω). Figure 1a shows three possible de-spinning histories with a_0=17.5R_e but different b (i.e., 0.2, 0.4 and 0.6) in which Ω decreases from ~6.5Ω_p (i.e., ~4.6 days per month) at t=0 to its present-day value Ω_p.

In solving the governing equations with time-dependent viscosity and spin rate, we first discretize the Maxwellian rheological equation in time [9], and then transform governing equations to a matrix form for a given spherical harmonic degree and time step. A propagator matrix technique is used to solve for the deformation and stress at each time step, using solutions from the previous time step or initial conditions. Time-dependent rotational force and viscosity are incorporated in the propagator matrix form. This method is similar to that used for surface loading problem of basin relaxation [13]. We verified the method by comparing solutions with those from a finite element method [9].

Results and Discussions: We first compute bulge size as a function of time in response to the spin rate histories in Figure 1a for a "fluid" model. The "fluid" model uses a uniform viscosity of 10^{22} Pas everywhere including the surface (i.e., no lithosphere). For this viscosity structure, the Moon would behave like a "fluid" planet over a time-scale of several Maxwell times (i.e., ~10^4 years), and the hydrostatic theory is expected to correctly predict the bulge size at any given time. The dashed curves in Figure 1b show the model bulge sizes at different times over the last 4 Ga that are normalized by the predicted present-day bulge size from the hydrostatic theory. The bulge size decreases with time as spin rate decreases. As expected, the normalized bulge size at the present-day is 1 for all the cases for the "fluid" model. In fact, the bulge size at any time is in agreement with the predicted from the hydrostatic theory. Note that the normalized bulge size for the observed is between 7 (for degree l=2 and order m=2) and 22 (for l=2 and m=0), and we take ~20 as a representative value for the rotational bulge that we compute here.

The bulge develops very differently when the temperature-dependent viscosity (i.e., lithospheric thickening) is included in the model. For all the cases, the normalized present-day bulge size is greater than 1 (Figure 1b), indicating that the bulge size is larger than predicted from the hydrostatic theory and that the lithospheric thickening leads to a fossil bulge. However, for a given initial semi-major axis a_0, a smaller b or more rapid de-spinning (or increase in a) in the early time leads to a smaller fossil bulge. This is because it takes time for lithosphere to thicken sufficiently to be able to maintain a fossil bulge. In our model, this time scale is controlled by the thermal diffusion that cools the Moon and thickens the lithosphere. For example, it takes ~300 Myrs for high viscosity effectively elastic lithosphere to thicken to ~150 km. For a_0=17.5R_e, b ranging from 0.4 to 0.6 is needed to produce a fossil bulge that is consistent with the observed (Fig. 1b). With this range of b, significant change of a or Ω occurs over time scales of 300-600 Myrs (Fig. 1a). Cases for different initial semi-major axis a_0 are also computed. A too large a_0 may not reproduce the observed bulge, regardless of b. A smaller a_0 can accommodate a smaller b or a shorter time scale to generate the observed bulge. Our calculations therefore provide a parameter space in a_0 and b (or time scales for early orbital evolution or Q value) that would explain the observed bulge.

In summary, we formulated a physical model for development of the lunar fossil bulge. This model indicates that for a reasonable lithospheric thickening history, the early lunar orbital behavior may evolve on a relatively long time scale (~300 Myrs or longer). This suggests that the early Earth may have a relatively large Q, and this has implications for the timings of the surface ocean and/or a deep magma ocean of the early Earth. Additionally, the lithospheric thickness needed to maintain the bulge may have implications for understanding small-scale gravity anomalies such as mascons.

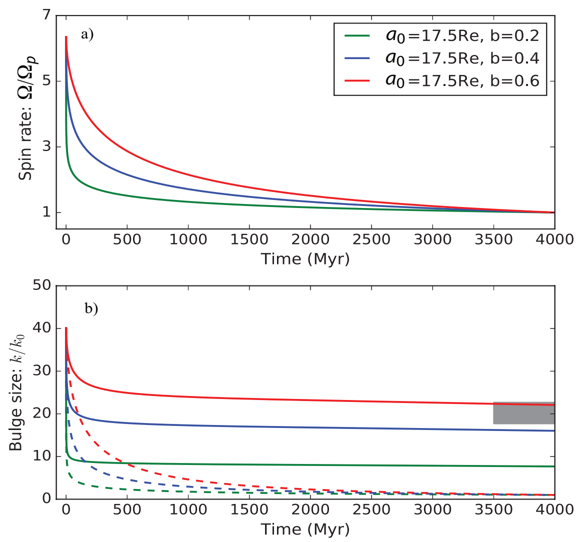

Figure 1. Normalized spin rate (a) and bulge size (b) vs time for three different de-spinning history. The dashed curves in (b) are for "fluid" cases. The shaded box represents the observed fossil bulge size.

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