Orientation statistics and error analysis of best-fitting planes from remote-sensing data

The orientations of planar rock layers are fundamental to our understanding of structural geology and stratigraphy. Remotely-gathered bedding orientations from photogrammetric terrain models underpin foundational studies of sedimentary and structural processes on early Mars. With the advent of unmanned aerial vehicles (UAVs), this technique is increasingly used to supplement outcrop measurements in Earth science as well.

Minimization algorithms typically used to fit planes to point clouds do not provide means of quantifying, but the uncertainty of the fit could still be high depending on the scale of the input datasets. We seek to quantify the absolute statistical power of these measurements.

A principal-components based method for fitting orientations

We develop a statistical method based on principal-component analysis (PCA), which has been used for planar fitting in palaeomagnetism and robotics. PCA minimizes orthogonalized regression lines, removing the implicit null assumption of orthogonality inherent in linear least-squares fitting. PCA rotates data into alignment with orthogonal axes of maximal variation of the dataset. The first two principal components, $x_1$ and $x_2$, define the fitted plane, with errors oriented along $x_3$. This representation is convenient for visualization of sources of error within the plane. PCA is not usually employed within a probabilistic framework, reflecting its main use case as a dimensionality-reduction tool in exploratory data analysis. Fitting planes to 3D point clouds requires no dimensionality reduction: the data remains tied to Cartesian spatial coordinates. The approach is equivalent to orthogonal regression and is amenable to probabilistic consideration.

Statistical analysis

The magnitude of error to the fitted plane is captured by the equivalent representations $\hat{M}$ and $\tilde{M}$.

2 Rotation into PCA-aligned coordinate space

The eigenvectors $\hat{V}$ define the orientation of the fitted, establishing a mapping from the Cartesian basis to a set of axes $\hat{V}$ aligned with the principal components of $M$. The eigenvectors are equivalent to the variance of the decorrelated data along those axes: $\lambda = \hat{V}^T M \hat{V}$.

3 Error analysis in PCA-aligned coordinate system

In Gaussian statistics, the error of each eigenvector can be represented as

$$\sigma^2 = \frac{\lambda}{n(n-1)} \text{ tr}(V^T U V)$$

where $n$ is the number of data points in $M$. An alternative representation using the “noise variance” of the input data, $\sigma^2 = \text{tr}(V^T U V)$, tailored specifically to principal components:

$$\sigma^2 = \frac{\lambda}{n(n-1)}$$

Error bounds at a given confidence level ($\alpha = 0.95$) are constructed for each axis using the Fisher test statistic for $\alpha = 2$ degrees of freedom and $n$ data points:

$$\delta = F_{\alpha, n-2} \cdot \sigma^2$$

These errors can be converted to the axes of a hyperboloid representing the maximal error surface.

$$v = [v_1, v_2, v_3]$$

These bounds define the error bounds on the fitted plane.

4 Projection into spherical coordinates

For an axis-aligned and mean-centered conic, the hyperboloid error bounds over the unit sphere $U$ are given by the equivalent representations $\sigma = \hat{V}^T M \hat{V}$.

Dataset variance as regression error

In standard regression analysis, the plane is fit through the mean of the dataset, and errors converge towards 0 for large sample sizes, even in the presence of significant data variance. This can lead to false certainty in over-sampled cases.

Since we suggest that our data should be fit on a plane, defining the errors to the plane with reference to the “true value” of the dataset allows errors to converge to the scale of deviations from a single plane. This provides a more precise statistical tool for noisy plane measurements.

Translation to spherical errors

Error analysis within an angular strike/dip frame is inherently problematic due to nonlinearities arising from the spherical basis. Error analysis of strike and dip relies implicitly on the small-angle assumption, and strike and dip errors expand nonlinearly when approaching zero dip, complicating error analysis of near-horizontal bedding.

Conclusions

This new mechanism for fitting bedding orientations allows the propagation and visualization of errors in remotely-sensed bedding orientations. The method pushes the limit for geologic inference towards the resolution limit of surface modeling, and broadens the set of applications that can be made from heterogeneous datasets of aerial and orbital bedding-orientation measurements. Going forward, we hope it will support a new generation of nuanced stratigraphic studies on Mars.

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References


