Binary Asteroid Orbit Sensitivity to Gravity Field Coefficients: Applications to the AIDA Mission Target 65803 Didymos.

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Summary

This study seeks to provide a baseline correlation between gravity field coefficient expansion order and the dynamical behavior of the system. To accomplish this we:

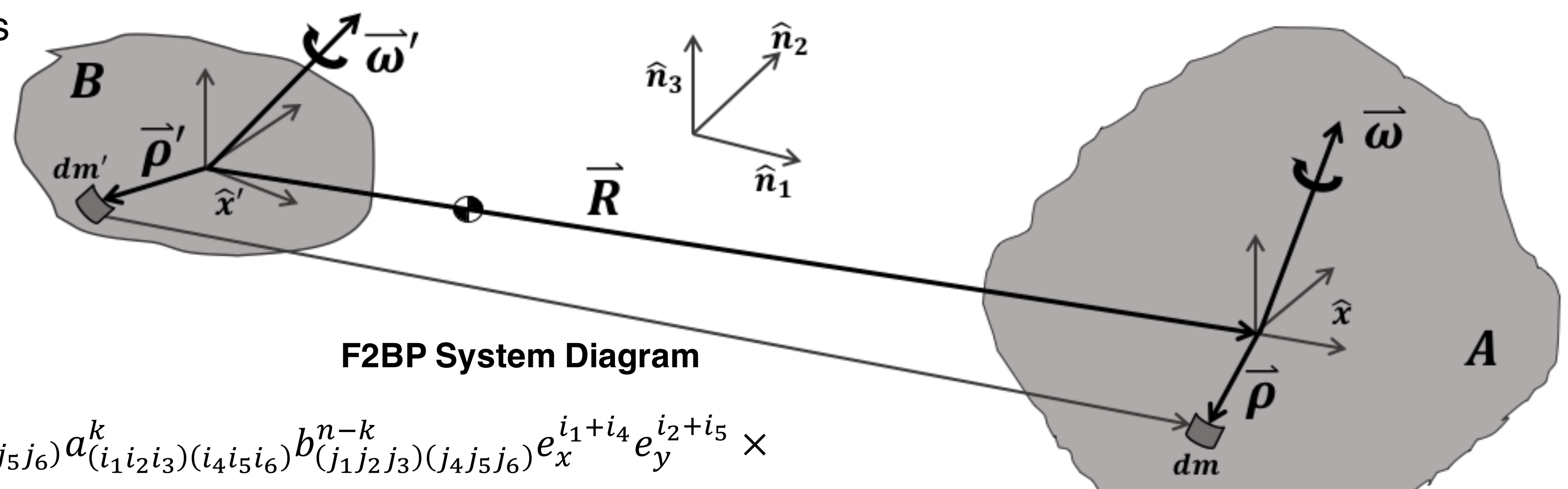
- **Implement an arbitrary shape and expansion order full two-body problem (F2BP) dynamics model.**
- **Analyze the simulation accuracy of increasing orders of mass distribution expansions through inertia integrals.**
- **Measure dynamical effects of mass parameter perturbations on primary and secondary asteroid models.**

Mutual Gravity Potential and Inertia Integrals

The implementation of inertia integrals for the mass distribution models enables a recursive formulation of the mutual gravity potential between the two bodies.

$$T_B^{ijk} = \frac{1}{M_B r^{i+j+k}} \int x^i y^j z^k \, dm \quad [1]$$

$$U = -G \sum_{n=0}^{N} \frac{1}{R^{n+1}} \tilde{U}_n \quad [2]$$



F2BP System Diagram

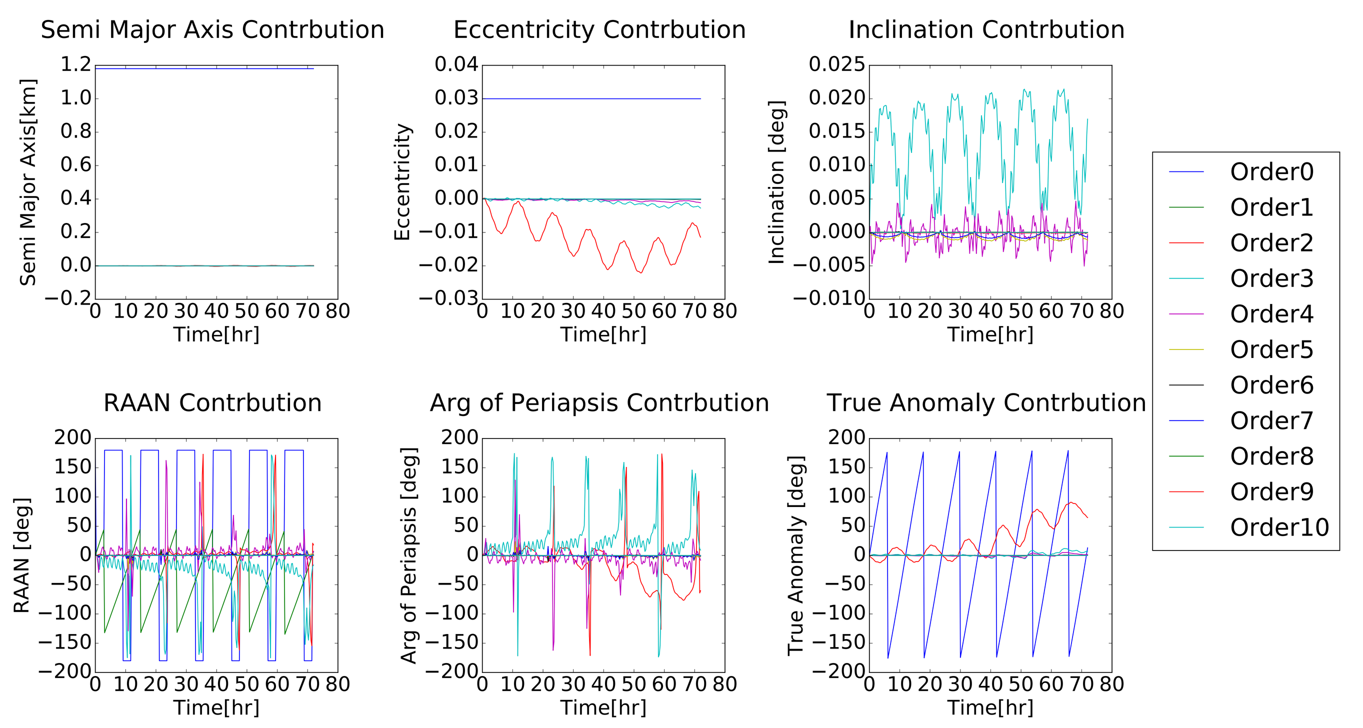
$$\tilde{U}_n = \sum_{k(2)=n}^{n} t_k^n \, \Sigma_{(i_1i_2i_3)(i_4i_5i_6)(j_1j_2j_3)(j_4j_5j_6)} a^k_{(i_1i_2i_3)(i_4i_5i_6)} b^{n-k}_{(j_1j_2j_3)(j_4j_5j_6)} e_x^{i_1+i_4} e_y^{i_2+i_5} \times e_z^{i_3+i_6} M_B R^{i+j+k} T_B^{(i_1+j_1)(i_2+j_2)(i_3+j_3)} M_{B'} R^{i+j+k} T'^{(i_4+j_4)(i_5+j_5)(i_6+j_6)}_{B'} \quad [3]$$

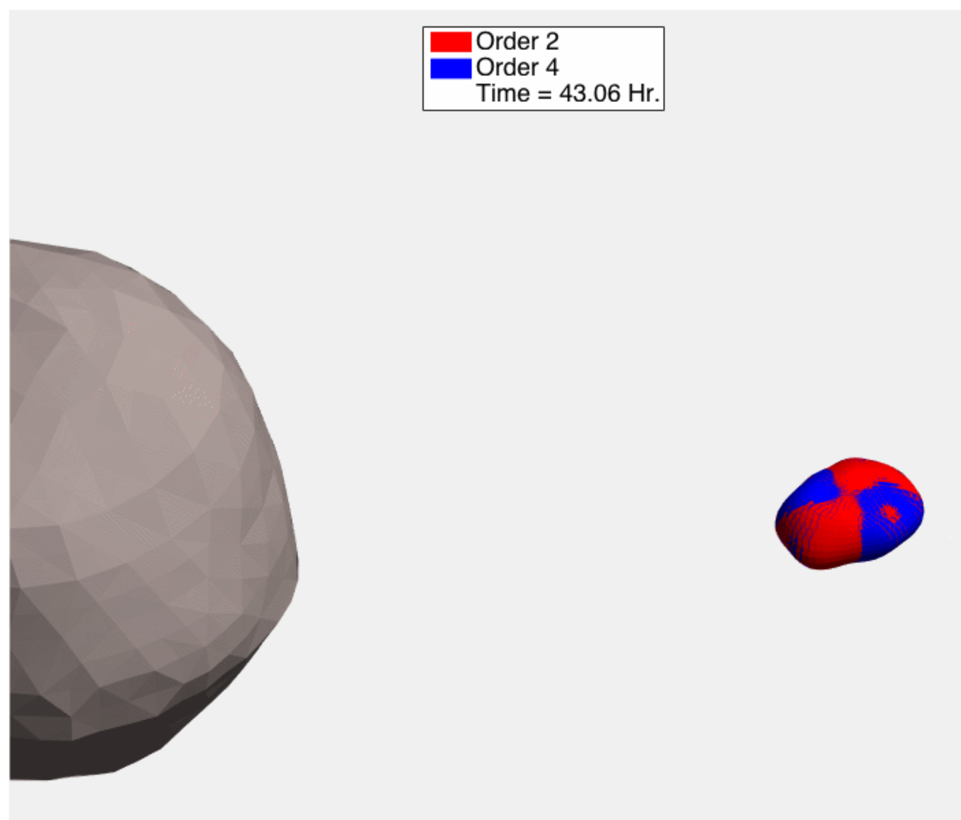
where k(2) implies stepping by 2

Effects of Expansion Order on Simulation and Observation Accuracy

Simulations of the Didymos nominal state for six orbit periods (~72 hours) were performed for expansion orders 0 through 10. Through these comparisons a relationship between dynamical accuracy and mass distribution accuracy can be developed on a per orbit basis.

- **Orbital motion converges to centimeter level accuracy by order 4, with meter level errors at order 2. Attitude shows slower convergence but reaches similar levels of accuracy between order 4 and 6.**



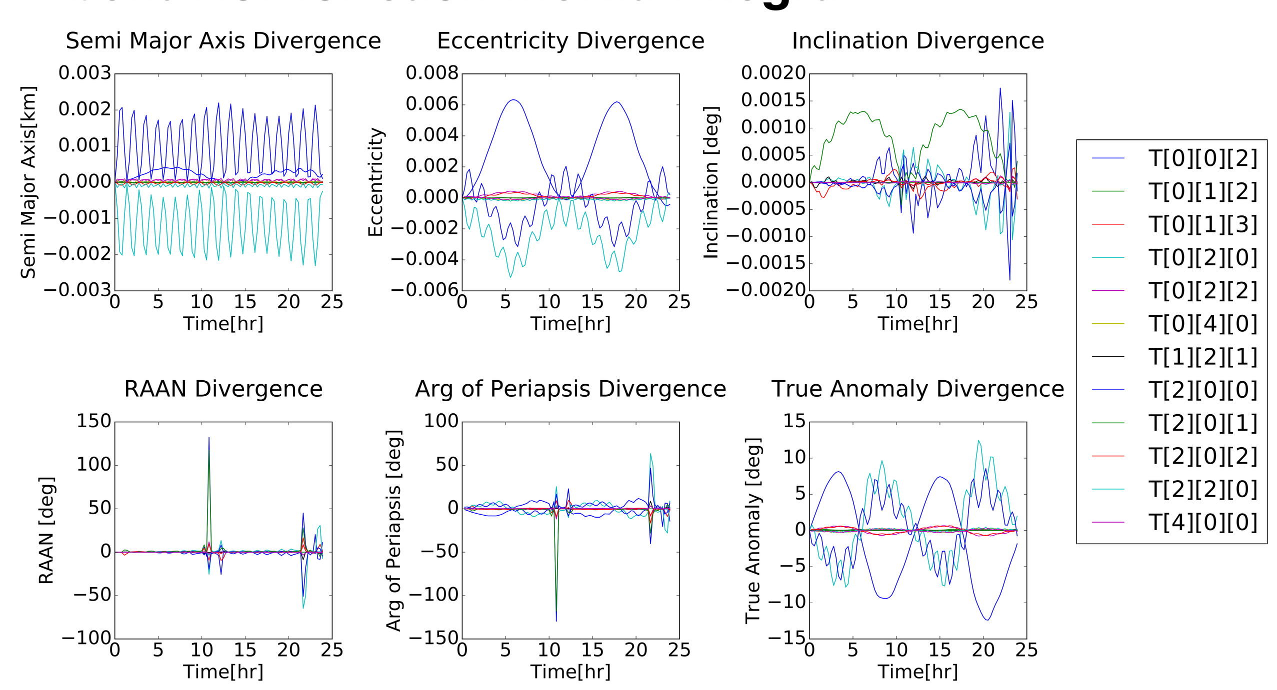


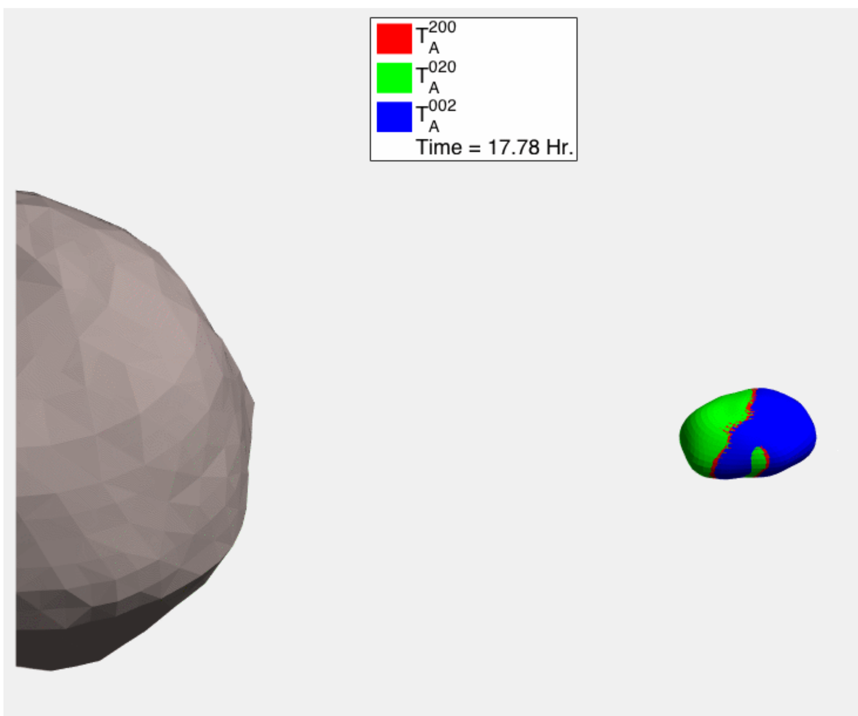
Contribution of inertia integral order to Kepler elements (top) and order 2 vs. 4 attitude visualization (bottom).

Effects of Inertia Integral Perturbations On Mutual Dynamics

To develop a baseline understanding of modeling and estimation errors on the mutual dynamics order 0, 2, 3, and 4 Inertia Integrals are perturbed by 10% of nominal value and investigated independently.

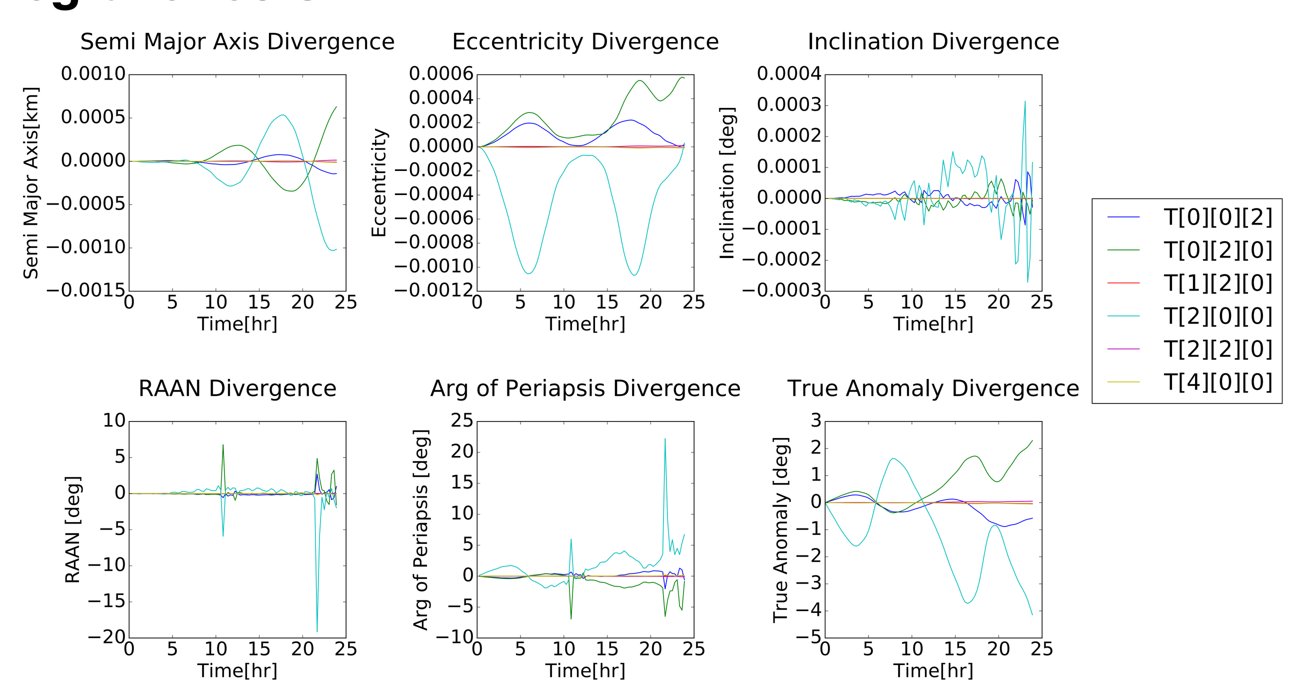
- **Effects of primary asteroid perturbations are primarily oscillatory and not secular, but do show unique behavior for each inertia integral.**

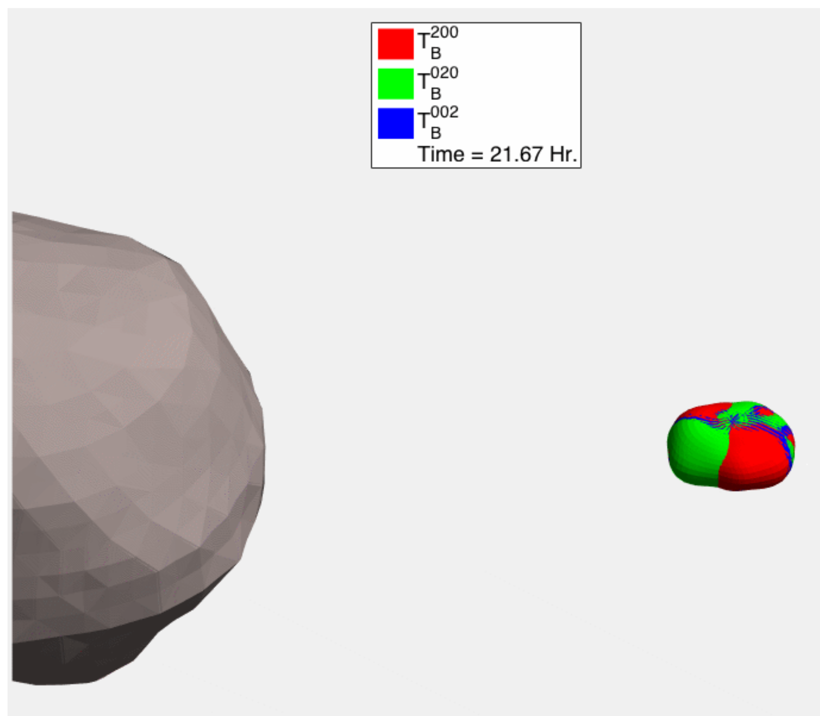




Divergence from nominal Kepler elements by each perturbed primary inertia integrals (top) and order 2 primary perturbation attitude visualization (bottom).

- **Effects of secondary asteroid perturbations show very secular motion and distinct behavior between inertia integral effects.**





Divergence from nominal Kepler elements by each perturbed secondary inertia integrals (top) and order 2 secondary perturbation attitude visualization (bottom).

Discussion

- **Simulation convergence shows meter level accuracy for 2nd order mass distribution models and cm accuracy for 4th order for the Didymos system.**
- **Preliminary studies show that perturbations of asteroid mass distribution parameters show up as secular and periodic deviations from nominal solutions, indicating the potential for refining estimates of them.**
- **As a result mass model errors and asymmetries up to 4th order may be detectable for the AIDA mission.**
- **Future Work: Development of analytical perturbation models of mutual dynamics and covariance analyses for mass distribution estimation.**

Acknowledgments

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