CRATER EQUILIBRIUM AS AN ANOMALOUS DIFFUSION PROCESS. D. A. Minton[1] and C. I. Fassett[2], [1]Purdue University, Dept. of Earth, Atmospheric, & Planetary Sciences. 550 Stadium Mall Dr., West Lafayette, IN 47907. (daminton@purdue.edu) [2]Mount Holyoke College, 50 College St., South Hadley, MA 01075

Introduction: Crater equilibrium (or crater saturation), is observed or suspected on cratered terrains throughout the solar system, yet it is still a poorly-understood phenomenon. Conceptually, cratering equilibrium is simple: when a surface becomes so heavily cratered that the addition of each new crater of a particular size erases, on average, one old crater of the same size, then the surface is said to be in equilibrium for that size crater [1]. Gault [1] found from both lunar observations and laboratory experiments that equilibrium occurred when the density of craters of a certain size reached 5-10% of that achieved by efficient packing of circles (geometric saturation). This implies a cumulative crater size-frequency distribution (SFD) in equilibrium of $N_{>D} = 0.0355D^{-2}$, where $N_{>D}$ is cumulative number of craters per km^2 and D is crater diameter in km. Hartmann [2] used observations of the Moon, Mars, and Mercury to develop an "empirical saturation" formula given by $N_{>D} = 0.0468D^{-1.83}$. Both of these "empirical saturation" lines are shown in Figure 1.

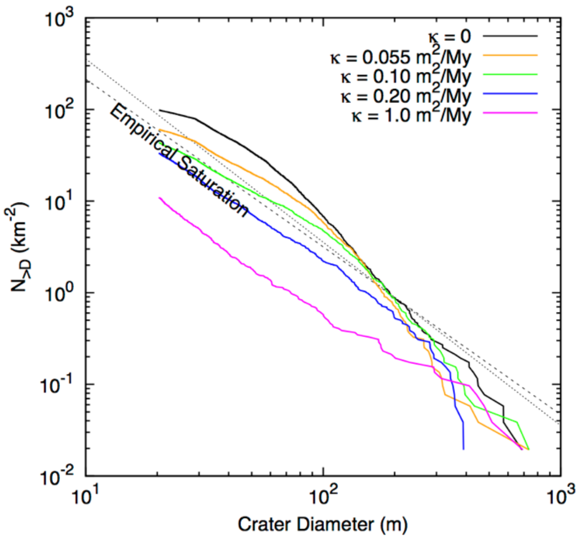

Figure 1: Effect of global diffusion on crater counts. Each solid line is a cumulative crater SFD generated by CTEM on a surface at 7.2 m/pix resolution. Each simulation is identical except for the value of the global topographic diffusivity. The dashed black line is the empirically-determined equilibrium line of Gault [1] and the dotted black line is the equilibrium line of Hartmann [2].

Attempts to understand saturation using Monte Carlo cratering codes have generally begun with the assumption that the equilibrium level is a result of a combination of cookie cutting obliteration (overprinting of craters by other craters) and sandblasting effects (erosion of large craters by small craters) [3-5]. Here we revisit the problem of crater equilibrium using the Cratered Terrain Evolution Model (CTEM) [6, 7].

Super-equilibrium crater counts: The CTEM code simulates the topography of a cratered terrain [6]. It uses a human-calibrated counting algorithm to determine the SFD of visible craters on the surface [7]. Crater erasure by ejecta burial is well constrained based on a model for the ballistic emplacement of ejecta. We performed an experiment to crater a lunar surface at with a resolution of 7.2 m/pix for 3.5 Gy. The SFD of the countable craters is shown as the solid black line of Figure 1. We confirm the result of Hartmann & Gaskell [4] that the crater counts achieve densities well over "empirical saturation" if no additional source of topographic diffusion is included.

Sub-pixel crater contribution: Hartmann & Gaskell [4] included a parameterized sandblasting model into their Monte Carlo code, which was intended to account for the erosive effect of sub-pixel craters (that is, craters smaller than the simulation resolution). Here, we investigate this hypothesis by developing our own model for sub-pixel craters using topographic diffusion.

To begin, we must calculate the effective diffusivity of each crater. Topographic diffusion is given by the formula [8]

$$\frac{\partial h}{\partial t} = \kappa \nabla^2 h$$

where h is the elevation, and κ is the diffusivity. We performed a series of numerical experiments in which we restricted our production SFD to narrow ranges of diameter in order to determine the contribution to the diffusivity κ from craters of a given size.

The diffusivity in these experiments is related to the cratering rate, which depends on the production function. In order to remove this dependency, we normalize the effective diffusivity by the incremental cratering rate, $/\dot{N}_\iota$. The units of this quantity are m^4, and it may be thought of as the diffusive "power" of each crater. We find that $\kappa/\dot{N}_\iota \approx 0.1D^4$, where D is the crater diameter. We can use this relationship to estimate the total diffusivity contributed by all craters below a simulation's resolution, provided that we can estimate the slope of the production SFD.

We combine our expression for $\kappa/\dot{N}_\iota$ with the Neukum Production Function (NPF) [9] to estimate the total cumulative value of the diffusivity, κ, of all craters smaller than a given size. The result is shown as the solid black line in Figure 2. We have also plotted in Figure 2 the range of κ values obtained by a study of the degradation state lunar mare craters by Fassett & Thomson [8].

We also performed a series of numerical experiments in which we applied a constant diffusivity to our test grid during the simulation in order to simulate the effect of sub-pixel cratering. We ran identical simulations of cratering in order to determine the value of κ needed to reproduce empirical saturation. An example of one of these experiments is plotted as color lines in Figure 1. The resulting values of κ calculated in these experiments for two different pixel resolutions, 3.6 m/pix and 7.2 m/pix, are plotted in Figure 2.

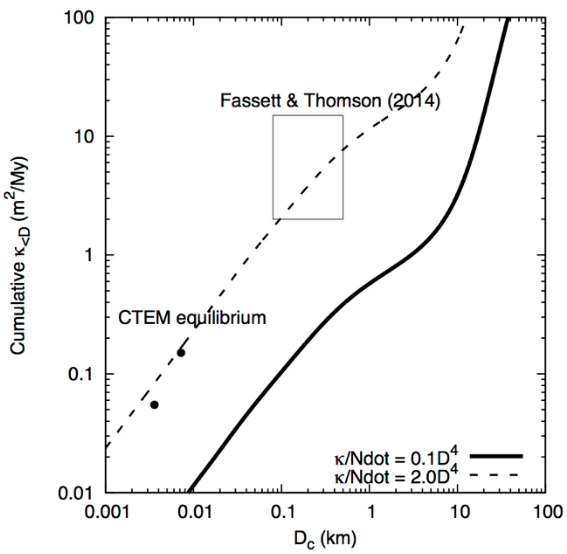

Figure 2: The solid black line is the estimated cumulative diffusivity, κ, for all craters smaller than diameter D_c, which is found by applying the crater diffusivity model to the NPF . The box shows the estimated range of values for κ obtained from observations of the degradation of 800 m-5 km craters on the lunar mare by Fassett & Thomson [8], assuming that degradation of a given crater is due to craters ≲10% its own diameter. The points show values of κ needed to reproduce the empirical equilibrium line from CTEM simulations like those shown as the color lines in Figure 1. The dashed line is that obtained using a model for crater diffusivity where $\kappa/\dot{N}_\iota \approx 2.0D^4$.

Conclusion: Figure 2 shows that our model under predicts the diffusivity of craters by a factor of ~20, but that the D^4 dependence appears to be supported by both our numerical crater equilibrium experiments and the Fassett & Thomson [8] study of lunar crater degradation. This strong dependence on diameter means that the cumulative effect of cratering is not topographic diffusion with a constant value of κ, as was assumed in that study. Instead, each crater size in the production SFD is associated with its own effective influence on the aggregate diffusivity. As a result, craters and other landforms at different spatial scale experience distinct diffusive forcing (i.e., different effective diffusivities). Although Fassett & Thomson [8]'s estimates for the diffusion rate remain applicable for ~kilometer-scale craters, the extrapolation of the diffusivity obtained in that study to the evolution of smaller craters is incorrect. The landform evolution experienced by the Moon is not classical diffusion; instead, the type of diffusive behavior on the lunar surface is an anomalous diffusive process.

Both classical and anomalous diffusion are related to a random walk process. In classical diffusion, the mean square displacement of the system is linear with time, while in anomalous diffusion the means square displacement has a power law dependence on time [10]. The random walk of material on the lunar surface is primarily controlled cratering, and therefore displacement distance of material by any cratering event depends on the crater's size, which is described by the nearly power-law production SFD. The mixing of material on the lunar surface is a similar process, and has been well-characterized using anomalous diffusion models [11].

This anomalous diffusion behavior is likely the reason why the oldest craters on any particular terrain appeared to have a higher value of κt in [8] than expected. On older terrains, larger craters, with larger effective κ, are progressively more important, so that κ should increase with time, even in the absence of changes in the cratering rate. Correctly accounting for this effect when the impact flux was in fact higher during the Late Heavy Bombardment will be important to understand crater degradation in the lunar highlands.

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