THE SPATIAL FRACTAL DIMENSION OF CRATERS ON LUNAR AND MARTIAN SURFACES. Weijie Xu[1], Bo Li[1], Zhongchen Wu[1], Zongcheng Ling[1], Jiang Zhang[1]. 1Shandong Provincial Key Laboratory of Optical, Astronomy and Solar-Terrestrial Environment; Institute of Space Sciences, Shandong University, Weihai, China, 264209. (xuweijiehaha@126.com).

Introduction: Fractal dimension (FD) is the main tool to describe the complex structures of different fractal parts, which was proposed by Mandelbrot [1]. FD can be used to describe the morphologic features on lunar and martian surfaces, because of its advantages in scale invariance and self similarity, etc. [2, 3].

Method: In this paper, we estimate the fractal characteristics of craters' distributions through two parameters: Box-counting Dimension and Information Dimension.

Box-counting Dimension (DB). The box-counting method involves covering an object with boxes of different side lengths and counting the number of boxes occupied by the object. Supposing the length of box is ε, the number of occupied boxes $N(\varepsilon)$ is counted for each box [4]. The DB value can be calculated by obtaining the slope of the linear logarithmic regression line:

$$DB = \lim_{\varepsilon \to 0} \frac{lg\, N(\varepsilon)}{lg\, \varepsilon} \qquad (1)$$

Information Dimension (DI). First, numbering the boxes from 1 to $N(\varepsilon)$, then calculating probability P_i that craters falling into the box i. Lastly, we can compute the DI when the box side length is ε:

$$DI = -\sum_{i=1}^{N(\varepsilon)} P_i \ln P_i \qquad (2)$$

In the box-counting method, the identified craters are covered with boxes of many different sizes. Sharma and Byrne [5] think the box-counting method is less precise and reliable compared to the ruler method because a spherical planetary surface cannot be divided into many equal-area boxes easily considering the projection. In this paper, we use a icosahedron with 20 equal-area triangles to simulate the surface of a planet. Then we subdivide every triangle to get four new triangles show in fig.1. The newly introduced vertices lie slightly inside the sphere, so push them to the surface by normalizing them. This subdivision process can be repeated for arbitrary accuracy and the divided results by boxes (from -60 °to 60 °) are show in Fig.1.

Results and Discussions: Craters are a topographical landmark or tectonic units on the planetary surfaces. The spatial distribution and the number of craters in different planets are different, thus there are various DBs and DIs in lunar and martian surfaces.

Here the craters' information are downloaded from the lunar crater database [6] and martian crater database [7]. Firstly, we calculate the spatial DB and DI of craters on global, maria and highlands separately (Table.1).

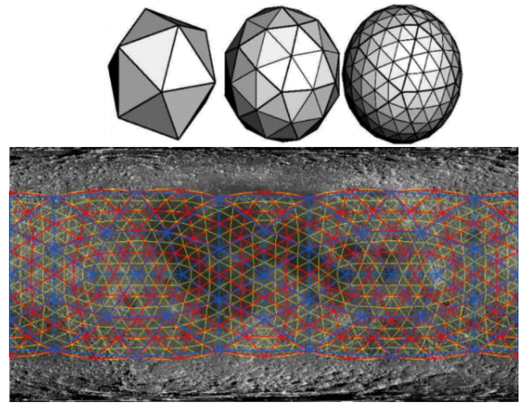

Fig.1. The three objects using 80,320,and 1280 (blue, red and yellow lines) approximating triangles, respectively.

From Table.1, we can see that the global craters' distribution has the highest DB, and maria has the smallest DB. Maria are formed by impact events that lead to basalt eruptions and deposits, thus, many pre-craters were eroded or covered by basalts. There are less craters in maria than in highlands which results in a lower DB. For DI values, the global, maria and highlands almost have same DI (0.235).

Table. 1. The DB and DI values of craters' distributions in global, maria and highlands

Region	DB	DI
global	1.67	0.2335
maria	1.25	0.2353
highlands	1.63	0.235

According to the craters diameters, we divided craters into three groups which are GA [1km≤D≤10km], GB [10km≤D≤100km] and GC [D≥100km]. We calculated the FDs of the three groups in global, maria and highlands (Table.2 and Fig.2). From Fig. 2, we can see that GA has the biggest FD values, GC has the smallest FD values. With the increase of the diameter, the fractal characteristics of craters distributions become obscure (from GA to GC), because that the several points on the left do not fall on the linear regression line (Fig.2.d).

In addition, we computed the DB and DI values of craters in seven impact basins (Table.3). We can find that: (1) craters in South Pole Aitken Basin and Mare Frigoris has the biggest and smallest DB values separately; (2) craters in Mare Crisium and Basin Orientale has the biggest and smallest DI values, (3) these impact basins have various FDs, this may relate to their different formation ages.

Table.2. The DB and DI values of craters' distributions of global, maria and highlands with GA, GB and GC

	Group	DB	DI
global	GA	1.63	0.20
	GB	1.26	0.19
	GC	0.34	0.07
maria	GA	1.20	0.25
	GB	0.56	0.12
	GC	0.29	0.09
highlands	GA	1.57	0.23
	GB	1.26	0.2
	GC	0.27	0.07

Table.3. The DB and DI values of craters' distributions in maria and basins

Mare	DB	DI
Mare Crisium	1.1291	0.5056
Mare Imbrium	1.3589	0.4043
Mare Tranquillitatis	1.0979	0.3652
Mare Frigoris	1.0431	0.361
Oceanus Procellarum	1.3332	0.332
South Pole Aitken Basin	1.6487	0.3163
Basin Orientale	1.2143	0.3055

According to topographic landscapes on Mars, the martian surfaces can be divided into three units [8]: I (the northern plain), II (the southern plateau), III (the midwest highland), respectively. We calculated the spatial FD values of craters on global martian surface and these three units. From Table. 1 and Table. 4, we can find that DB (1.875) and DI (0.246) in global Mars higher than DB (1.670) and DI (0.233) in global Moon. The II region has the highest DB (1.84), while the III region has the lowest DB (1.636). For the DI, its trends are opposite to the DB, thus the III region has the highest DI (0.3062), the II region has the lowest DI (0.2568). This may be caused by the different geological and weathering, erosion processes affecting the martian surface.

Table.4. The DB and DI values of craters' distributions on global martian surface and region I, II and III

Region	DB	DI
global	1.87	0.25
I	1.68	0.28
II	1.84	0.26
III	1.64	0.30

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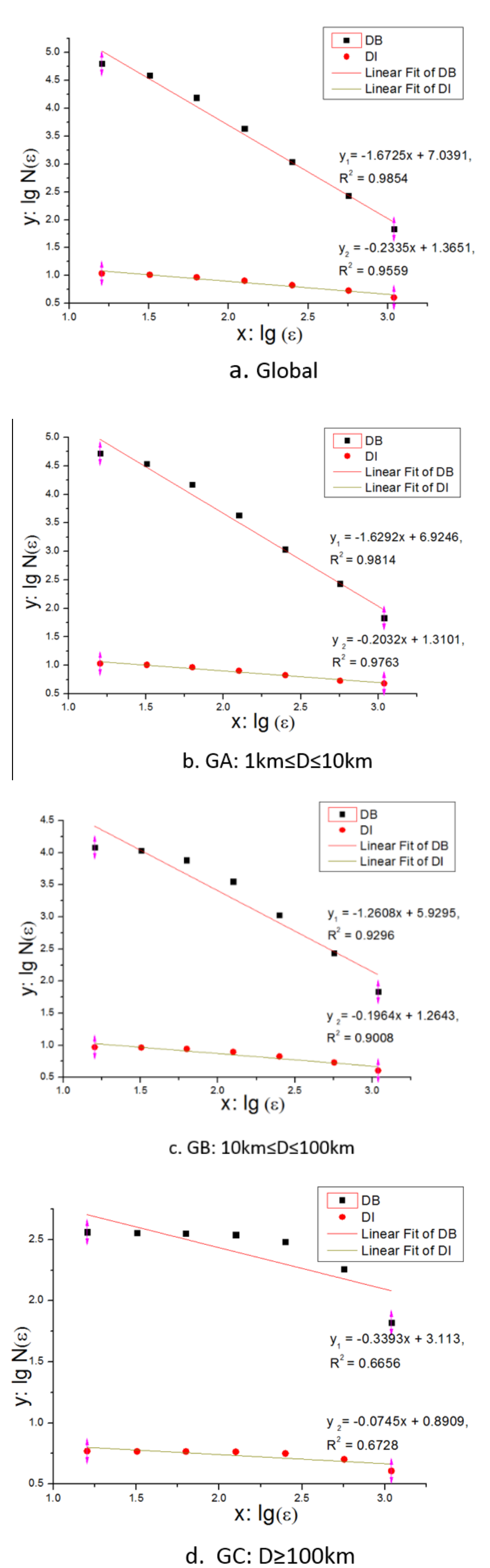

Fig.2. The FD values of craters' distributions on Moon and GA, GB and GC groups. y_1 stands for DB value and y_2 stands for DI value.