

INHOMOGENEOUS TWO-LAYER INTERNAL STRUCTURE AND MOMENTS OF INERTIA OF PHOBOS. K. Matsumoto¹ and H. Ikeda², ¹RISE Project Office, National Astronomical Observatory of Japan (Mizusawa, Oshu, Iwate, 023-0861 Japan, koji.matsumoto@nao.ac.jp), ²RDD/JAXA (Tsukuba, Ibaraki, 305-8505 Japan).

Introduction: The origin of Phobos is still an open issue. It may be either captured asteroid or formed from a disk of impact ejecta produced by a giant impact. Although it is not straightforward to determine the origin from internal structure alone, it will place important constraints. One of the key parameters related to the internal structure is moments of inertia (MOI). Phobos' MOI can be determined from amplitude of short-period forced libration (θ) and degree 2 gravity coefficients (C_{20} , C_{22}). Currently, the libration amplitude is estimated to be $\theta = 1.09 \pm 0.10^\circ$ from control point network analysis using multiple image data [1]. Although the degree 2 gravity coefficients are estimated from tracking data of Mars Express on its close flyby at Phobos, they are not solved for at sufficient accuracy [2]. Axial difference of MOI can be constrained by the libration amplitude, but currently MOI of Phobos is not known. The observed libration amplitude is consistent with homogenous mass distribution of Phobos, but local mass anomalies can not be ruled out [1,3].

Here we consider relatively simple two-layer internal structure and assume that ice water or porosity is confined in either layer, and calculate how much MOI deviate from the value for homogeneous body if such an inhomogeneity existed.

MOI from simple two-layer Phobos internal model: Phobos' bulk density of $1.86 \pm 0.013 \text{ g/cm}^3$ [4] is lower than most of the samples of carbonaceous material, which requires porosity and/or light elements like water ice. If the low bulk density was explained by water ice, its mass fraction is expected to be 10-35% depending on rocky material grain density [5]. If the mass distribution inside Phobos was inhomogeneous, e.g., water ice was concentrated near the surface or the center, we will observe a deviation of MOI from the value for homogenous interior. Here the MOI differences (ΔMOI) with respect to the homogenous Phobos are calculated for some cases where we assumed that (1) Phobos has a tri-axial ellipsoidal figure ($a = 13.03 \text{ km}$, $b = 11.40 \text{ km}$, $c = 9.14 \text{ km}$), (2) Phobos has a two-layer structure and their boundary also has the similar ellipsoidal figure for which the libration amplitude is 1.15 degrees being consistent with the observed value of [1], and (3) water ice is confined either of the upper or lower layer and rock density is the same for both the layers. The water ice mass fraction is changed between 0 and 30% .

Figure 1 shows the result for the case that upper layer is composed of the rock plus water ice. When the upper layer thickness is 10% of the semi-principal axes (labeled as $R_b = 0.9R$), no more than 14 wt.% of water can be contained in the layer and the maximum $|\Delta\text{MOI}|$ is about 9%. When the layer boundary is deeper, more water can be contained, but the maximum $|\Delta\text{MOI}|$ is about 16%. Shown in Figure 2 is the result for the case in which the water ice is confined in the lower layer. If the layer boundary is located below $0.6R$, no more than 11 wt.% of water can be contained in the layer and the maximum $|\Delta\text{MOI}|$ is about 8%. When the layer boundary is shallower, more water can be contained, but the maximum $|\Delta\text{MOI}|$ is about 17%.

We also tested the cases in which the porosity is responsible for the low bulk density. We calculated ΔMOI due to inhomogeneous distribution of the porosity using the similar two-layer structure. The results depend on the boundary depth and rock density. In the case that the lower layer is porous, the maximum $|\Delta\text{MOI}|$ is about 17% when rock density is 2400 kg/m^3 , and about 9% when rock density is 2100 kg/m^3 .

MOI from libration amplitude and gravity coefficients: The amplitude of forced libration in longitude for Phobos is expressed as (e.g. [1])

$$\theta = \frac{2e}{1 - \frac{1}{3\gamma}}$$

where e is the orbital eccentricity of Phobos and γ is dynamical flattening;

$$\gamma = \frac{B - A}{C}$$

where $A < B < C$ are MOI normalized by MR_0^2 (M : mass, R_0 : mean radius) along the principal coordinate axes. Since the libration amplitude is a function of difference in the MOI, it is difficult to determine each of A , B , and C from θ alone. However, by combining with degree 2 gravity coefficients which are also related to the normalized MOI as

$$C_{20} = \frac{B + A}{2} - C$$

$$C_{22} = \frac{B - A}{4}$$

each of MOI can be derived as follows (e.g. [6]);

$$A = \frac{\gamma C_{20} + 2(2 - \gamma)C_{22}}{\gamma}$$

$$B = \frac{\gamma C_{20} + 2(2 + \gamma)C_{22}}{\gamma}$$

$$C = \frac{4C_{22}}{\gamma}$$

Figure 3 shows error of the least MOI A as a function of errors of the libration amplitude and gravity coefficients, where C_{20} error is assumed to be the same as that of C_{22} . In order to realize a certain accuracy of MOI, it is required to achieve the similar level of accuracy for both the libration amplitude and the gravity coefficients, otherwise the MOI accuracy is dominated by either component with larger error.

Summary and perspectives: We study ΔMOI which is produced by inhomogeneous distribution of water ice or porosity using simple two-layer model for which the boundary has the similar shape as the surface. It is found that, in such a configuration, $|\Delta MOI|$ is smaller than 16-17%. A 10% accuracy will not be sufficient, and it is required to achieve at least a few percent of MOI accuracy in order to detect it. To this end, the required accuracies for the libration amplitude and the degree 2 gravity coefficients are also a few percent. This level of accuracy will be achieved by future Phobos missions. Our preliminary study on gravity recovery suggests that a precision of a few percent on degree 2 gravity coefficients is feasible when the altitude of spacecraft in a quasi-satellite orbit is as low as about 10 km. More of higher-resolution images of Phobos will also improve the estimate of the forced libration amplitude.

References: [1] Oberst et al. (2014) *Planet. Space Sci.*, 102, 45-50. [2] Pätzold et al. (2014) *Icarus*, 229, 92-98. [3] Rambaux et al. (2012) *Astron. Astrophys.*, 548, A14. [4] Willner et al. (2014) *Planet. Space Sci.*, 102, 51-59. [5] Rosenblatt (2011) *Astron. Astrophys. Rev.*, 19 (44). [6] Bills and Rubincam (1995) *J. Geophys. Res.*, 100, 26,305-26,315.

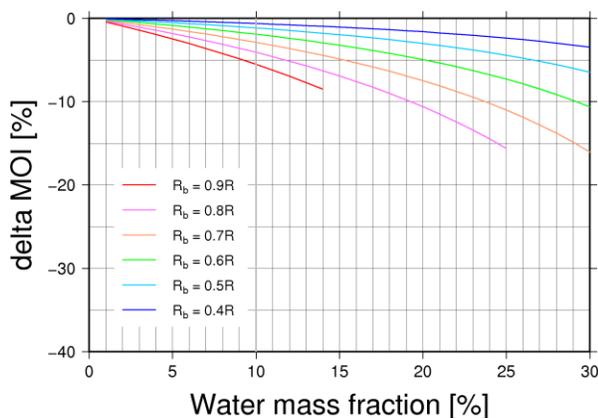


Figure 1. Moments of inertia differences between homogeneous and inhomogeneous Phobos. The inhomogeneity is assumed to be produced by water ice confined in the upper layer.

Different color indicates different location of the layer boundary (red: shallow, blue: deep).

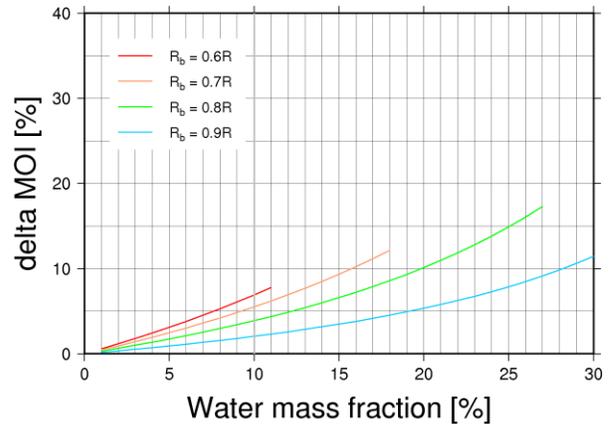


Figure 2. Same as Figure 1, but for the case that water ice is confined in the lower layer. Different color indicates different location of the layer boundary (red: deep, blue: shallow).

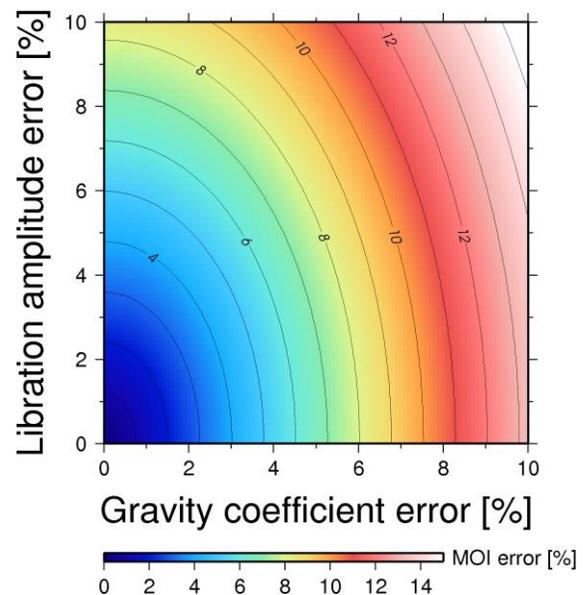


Figure 3. Relation of errors in the forced libration amplitude (θ) and the degree 2 gravity coefficients (C_{20} , C_{22}) to the moments of inertia error.