COMPARISON OF TWO METHODS ON LUNAR FREE LIBRATIONS CONSIDERING TWO-LAYER MODEL. W. X. Li[1, 2], A. Gusev[2, 3], and J. S. Ping[2], [1]Xinjiang Astronomical Observatories, CAS, China, liwenxiao@xao.ac.cn, [2]Lunar and Deep Space Exploration Division, National Astronomical Observatories, CAS, China, jsping@bao.ac.cn, [3]Russian Kazan Federal University, Kazan, Russia, Alexander.Gusev@mail.ru.

Introduction: Lunar librations have been researched for decades. We tried to compare the numerical and analytic methods on periods of lunar free librations. Furthermore, the layer structure has been assumed to be two layers and it corrected the periods a little from full-rigid Moon. However, this model is still inadequate for accurate research on lunar librations. There is still relatively big difference between current analytic and numerical results. We will try to locate the reason of the remained difference in the future work. Lunar librations supply a good opportunity for us to research the dynamical influence of lunar internal structures

Analytical Method: The physical librations come from the solutions of the rotational differential equations reduced by using the first Cassini law and linearized with respect to small quantities [1]. For rigid Moon, without considering liquid core and tidal influence, the linearized rotation equations at the first order of the Moon are basic equations to figure out the free librations. They are with respect to external forces, lunar mean motion and dynamical ellipticities, which are determined by principal moments of inertia ($A < B < C$).

The values of dynamical ellipticities are obtained from JPL ephemeris of DE430 [2].

Setting the external forces to be zero, we could obtain the eigenfrequencies of free librations by solving the equations. The low order of eigenfrequencies is the analytical frequencies of lunar free librations. Rambaux & Williams (2011) supplied them to us [1].

Using full Moon geophysical parameters, we could obtain the periods of free librations for rigid Moon.

Considering deeply, with two-layer structure, the periods would be corrected a little. In order to prove it, the parameters of lunar core are necessary. However, we need to find the relation of parameters of lunar core and full Moon.

While discussing the dissipation of lunar rotation, Williams gave us the relation between parameters of gravity field and dynamical parts [3].

Thus, geophysical parameters of DE430 supply the relation of rigid Moon and lunar fluid core [2].

However, it is still impossible to get the three values of moments of inertia of lunar core. Two simple assumptions on dynamical condition of lunar core have been made.

Assumption 1. If the fluid core is dynamically symmetric, there would be $A_c = B_c$.

Assumption 2. If the fluid core is non- dynamically symmetric, there would be $A_c \neq B_c$.

The further assumption is necessary. Following the dynamical assumption of Noyelles (2011) [4],

$$\frac{A_c}{A} = \frac{B_c}{B},$$

we could obtain the moments of inertia under this condition.

According to the relation between mantle and core, we could obtain all information we need on corrected rigid Moon. Then, analytically getting the frequencies and periods of free librations is possible.

Numerical Method: The lunar rotation combined forced and free librations. The numerical fitting method aimed at identifying the frequencies of free librations from forced part. The periods of lunar orbit is effected by solar planets [1].

The fit function, which is used to analyze the periods and amplitudes of librations, takes the following form [3]

$$f(t) = \sum_{j=1}^{m} a_j t^j + \sum_{i=1}^{n} \left[\left(C_i + \varepsilon_i^c t \right) \cos\left(\phi_i(t) \right) + \left(S_i + \varepsilon_i^s t \right) \sin\left(\phi_i(t) \right) \right]$$

where the coefficients are for polynomial fitting.

Based on above method, Rambaux and Willianms (2011) analyzed residuals for librations over 1070 years. They obtained the periods of free librations [1].

Results and Discussion:

Table 1 Analytical and numerical results

Periods (day)	Longitude	Latitude (MR)	Latitude (SR)	Wobble
Assumption 1	1044.92	27.2958	28852.72	27070.43
Assumption 2	1045.29	27.2958	28856.37	27069.46
Williams	1056.13	——	——	27257.27

Table 1 shows that lunar layer model could moderately correct the periods of free librations, but there is still relatively big difference between analytical and numerical results. The difference is about 10 days for longitude part and 200 days for wobble. The numerical method cannot identify coordinate reference for latitude, so this part cannot be compared. The difference

between analytical and numerical methods is about 1% for each part.

Conclusion: Analytical and numerical methods on free librations are relatively uniform, but there is still difference between them for accurate discussion. Considering layer model of the Moon, free librations would be corrected a little, but the two layer structure is inadequate for free librations.

In the future, we would furtherly locate the reason of the difference and figure out the influence of the uncertainty of geophysical parameters.

References: [1] Rambaux N, Williams J G. The Moon's physical librations and determination of their free modes [J]. *Celest Mech Dyn Astr*. 2011, 109:85-100. [2] Williams J G, Boggs B H, Folkner W M. DE430 Lunar Orbit, Physical Librations, and Surface Coordinates [R]. *JPL Interoffice Memorandum*, 2013. [3] Williams J G, Boggs D H, Yoder C F. Lunar rotational dissipation in solid body and molten core. *J. Geophy. Res*, 2001, 106: 27933-27968. [4] Noyelles B, Karatekin Ö, Rambaux N. The rotation of Mimas [J]. *A &A*. 2011, 536:A61.

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