NUMERICALLY SIMULATING OCEAN DISSIPATION IN THE ICY SATELLITES

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Introduction: Radiogenic heating and tidal dissipation are the primary ongoing thermal contributors to icy satellite interiors. Tidal dissipation may provide the means to form and support global subsurface oceans, such as those already confirmed in several of the icy satellites. Europa has been shown to have an induced magnetic field consistent with a global layer of salty, liquid water beneath its surface [e.g., 1, 2]. This is also the case with Ganymede and Callisto. Titan has been interpreted to have a conductive liquid layer beneath its surface due to the presence of a Schumann-like resonance [3]. Enceladus' forced libration also suggests a global subsurface ocean [4]

By studying and considering the effects of dissipation in the solid and liquid regions of the outer Solar System icy satellites, we hope to not only understand and help constrain their current thermal and interior structures, but also how the satellite's interior, rotation and orbit has evolved over time.

Tidal Dissipation: Despite the overwhelming evidence for subsurface oceans in multiple icy satellites, there are as of yet no studies that consider dissipation in a subsurface ocean that lies between two solid regions: the satellite's interior and rigid outer icy shell. Here we model ocean dissipation assuming no solid lid, and include deformation (but not dissipation) of the satellite's interior.

Assuming a surface ocean is a reasonable approximation to the Earth, where the majority of dissipation occurs in the fluid regions of the planet. However, the addition of a solid lid is expected to dampen the magnitude of ocean dissipation as the resulting tidal potential will be lowered due to the rigidity and internal strength of the overlying shell.

Tyler [6] and Matsuyama [7] developed an analytical formalism to model dissipation in icy satellite global surface oceans. We compare our numerical results to that from the model of [7] while ignoring the effects of ocean loading and self attraction.

Here we use our numerical model, Ocean Dissipation in Icy Satellites (ODIS), to investigate ocean dissipation using two friction models for Titan and Enceladus. The first model, Rayleigh friction, assumes that drag scales linearly with fluid velocity. This model has already been applied analytically [6,7]. Bottom friction is the second model investiagated, where drag scales with the square of fluid velocity, such as that experienced by a fluid parcel in a turbulent boundary layer near the ocean floor. Due to this square dependence, the model can only be solved numerically. Our results are compared to analytical solutions of [7] and [8] for verification and evaluation.

Numerical Model: Our numerical model is based on that developed by Sears [5]. Thus, we use a two dimensional finite difference computational fluid dynamics code to solve the mass (1) and momentum (2) Laplace Tidal Equations over a square grid [5]:

$$\partial_t \eta + \nabla \cdot (h\boldsymbol{u}) = 0 \tag{1}$$

$$\begin{aligned}\partial_t \boldsymbol{u} + 2\boldsymbol{\Omega} \times \boldsymbol{u} + \alpha \boldsymbol{u} + \frac{c_D}{h}|\boldsymbol{u}|\boldsymbol{u} + g\nabla\eta \\ = (1 + k_2 - h_2)\nabla U_2\end{aligned} \tag{2}$$

where η is the surface displacement from equilibrium, h is the ocean thickness, $\boldsymbol{u}$ is the tangential velocity, and $\boldsymbol{\Omega}$ is the angular velocity of the satellite. Ralyeigh and bottom friction coefficients are denoted as α and c_D respectively. g is the satellite's surface gravity and k_2 and h_2 are the degree-2 tidal Love numbers. The gradient of the tidal potential, U_2, is an applied force which is subsequently reduced or enhanced by Love's reduction factor, $1 + k_2 - h_2$ [5].

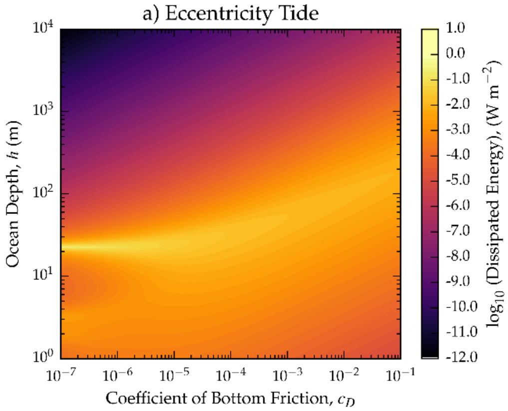
The numerical scheme is explicit and forward in time, and is outlined in several works by Zahel [e.g., 10]. We use a staggered grid in space and solve for the northward and eastward flowing velocity components, as well as ocean surface displacement η. From these quantities, $\boldsymbol{u}$ and η, ocean dissipation is derived, as in [5]. The model uses a grid spacing of 2-4 degrees in both latitude and longitude.

Methodology: Simulations were ran to find average ocean surface dissipation over the orbital period for the two main tidal components, eccentricity and obliquity, as a function of ocean depth and friction coefficient. Each tidal component requires 3000-5000 simulations in order to resolve any dissipative resonances in the system. These simulations are run for both Rayleigh and bottom friction separately, and for Titan and Enceladus in turn.

Results and Discussion: Comparison of the numerical and analytical results of [7] show mostly excellent agreement across all of the explored paratmeter space. The position of the resonances are well resolved, and for the deeper resonances the dissipation magnitudes show good agreement. Most of the observed discrepancy is a result of discretisation error, which we show to be reduced by increasing the model's spatial resolution.

Modelling with bottom friction tends to smoothen resonances over a broader parameter space. This suggests that for reasonable bottom friction coefficients (~ 0.002 [8]), dissipation lies towards the high Rayleigh friction coefficient regime, specifically around resonances (Figure 1). This high friction regime does not necessarily correspond to high dissipation as fluid velocity is heavily damped, reducing the amount of kinetic energy available for dissipation.

Interestingly, the Rossby wave resonance is now orientated vertically in this parameter space, restricting it to a narrow range of bottom friction coefficients.


a) Eccentricity Tide

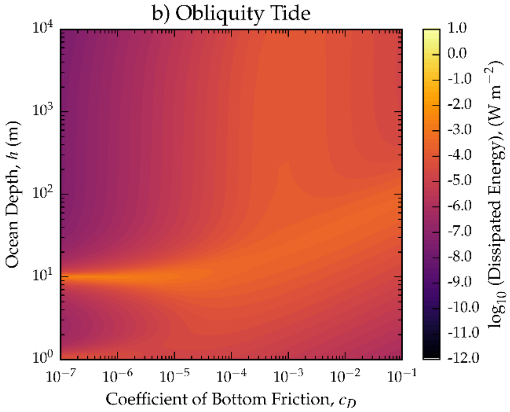
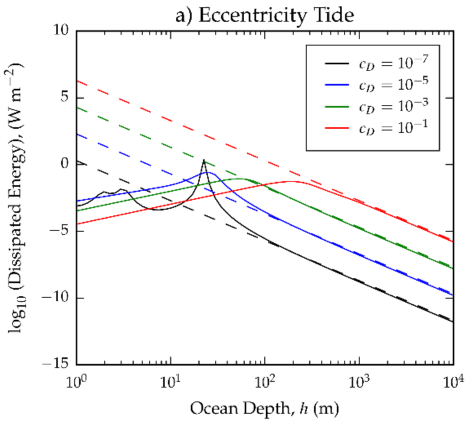

b) Obliquity Tide

Figure 1. Bottom friction ocean surface dissipation for the eccentricity and obliquity tides on Titan, averaged over the orbital period. The same gravity wave resonances from the Rayleigh friction case are still present [7], although the Rossby wave resonance is now localized to a specific range of bottom friction coefficient.

Away from gravity wave (horizontal) resonances and shallow oceans, our model has excellent agreement with scaling laws developed by [8] for quantifying bottom friction dissipation (Figure 2).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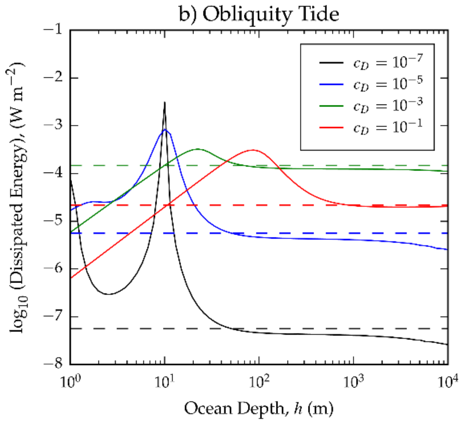
b) Obliquity Tide

Figure 2. Comparison of the numerical (solid lines) and scaling law (dashed lines) results developed by [8] for ocean dissipation using bottom friction on Titan. Results are in excellent agreement away from resonances and shallow oceans.

Conclusions: Our numerical model is shown to have excellent agreement with analytical solutions of ocean tidal dissipation in icy satellites, assuming a global surface ocean. Most observed error is a result of numerical discretisation.

We also simulate ocean dissipation using bottom, rather than Rayleigh, friction. Dissipative resonances are shown to be spread over a much broader range of ocean thicknesses throughout much of the parameter space in the bottom friction regime. This may correspond to the high Rayleigh friction environment. The Rossby wave resonance associated with the obliquity tide is also restricted to a narrow range of friction coefficient.

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