Angles of Repose of Granular Beds using a Soft-Sphere Discrete Element Method (SSDEM)

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Introduction:

During the last decade, SSDEM codes have been used to investigate the behaviour and evolution of granular asteroids and their surfaces as well as the interactions of the latter with man-made probes and spacecrafts. This codes simulate the motion of particles that interact through soft-potentials when they overlap and can reproduce the dynamics of real granular materials with certain limitations. Due to the complexity of simulating non-spherical shapes, most codes deal only with spherical particles. These particles feel a repelling forces that keeps the overlap to a minimum, and also surface-surface frictional forces; additionally, rolling friction attempts to mimic the effect of non-sphericity in their motion. This paper analyzes the effect of the coefficients of dynamic friction and rolling friction in the angle of repose of a granular bed in order to better understand their influence in our simulations. These and other numerical experiments are carried out as calibration runs so that we can find the realm of applicability of the simulation method and model parameters in the context of small planetary bodies, their surfaces and the upcoming sample retrieval missions to asteroids.

Simulation Method:

The particles in the SSDEM code that we have used [1, 2, 3] interact through soft potentials and these interactions are of three types: normal, tangential and rotational. The first produces normal forces that keep the particles apart when they overlap. The second is related to surface-surface friction and it is essentially a stick-slip interaction that satisfies the local Coulomb yield criterion. The third is similar in form to surface-surface friction, but it appears as a winding spring that opposes the relative rotation of particles in contact.

Normal Forces: In the SSDEM the motion of the particles is integrated numerically through the discretization of their equations of motion. The time step (δt) that is used for this integration depends on the material parameters of the particles; the harder the particles, the smaller the time step. During this time step, each particle is assumed to move under constant forces. When the particles overlap, a repelling force is applied at the contact point ($\mathbf{f_n}$). This force is a function of the overlap and, in spherical particles, it is applied along the line of centres of the overlapping particles. Usually, either linear or hertzian spring-dashpot models, with a elastic constant k_n, are used [4]. Multi particle contacts are allowed as well as enduring contacts.

Tangential Forces: (Surface-surface friction) When two particles are in contact, in a SSDEM code, they are allowed to slide. However, as frictional forces should oppose this motion, a truncated tangential spring-dashpot system must is implemented. As the initial points on contact in each particle move apart due to the latter's relative rotation, a spring is stretched. When the restoring force of this tangential spring reaches the value $\mu_k f_n$, the frictional force is not increased any longer so that the local Coulomb criterion is satisfied (μ_k is the coefficient of dynamic friction) [5].

Rolling and Twisting Friction: Rolling and twisting friction in a SSDEM code are used to mimic the behaviour of non-spherical particles. The implementation is similar to the surface-surface friction method, but instead of a tangential force that produces a torque on the particle, this kind of friction produces a torque that opposes rolling or twisting motion. This torque is a function of the particles sizes (reduced radius R_r), their moments of inertia, angularity and, more importantly, their relative rotation. The particles in contact are allowed to rotate relative to one another, but this winds a spring that produces a torque that opposes it. When this torque has reached the value $\mu_r R_r f_n$ where μ_r is the rolling friction coefficient, the torque is not increased anymore and the particle rotates [6].

Numerical Experiments:

For our experiments we have decided to run a series of simulations to find the influence of μ_k and μ_r in the dynamic and static angles of repose. The former is the maximum angle at which the granular bed can be tilted before the onset of an avalanche, the latter is the slope that results when the avalanche has stopped. In our simulations we use a granular bed formed by 30000 spherical particles with a size range between 6 and 13 mm. These particles have a density of 2500 kg m^{-3} and are contained in a box of 20x50 at its base and 60cm high. The particles that touch the base of the box are randomly distributed and glued to it. The bed itself has a height of $\approx$ 17 cm. Initially the particles are placed inside the box, given a random initial velocity and let fall under a 1g gravitational field, without friction. The value of μ_k and μ_r are varied between 0 and 1 in steps of 0.1. In total, we have ran 121 sets of parameters. Cohesive forces were not added in these simulations to limit the size of the parameter space.

To find the static angle of repose we need to incline a box with periodic boundary conditions. As studied by [7], friction between the particles and the walls can increase angle at which granular flow starts.

We mimic the inclination of the box by changing the direction of gravity; this change happens in steps of 3° every second. When the dynamic angle of repose has been reached, the particles flow continuously and do not stop. As a test, we also ran some simulations in which the inclination was continuously changed at a constant rate of 5° per second (not in steps); these simulations rendered results with greater precision when analyzed together with the original results.

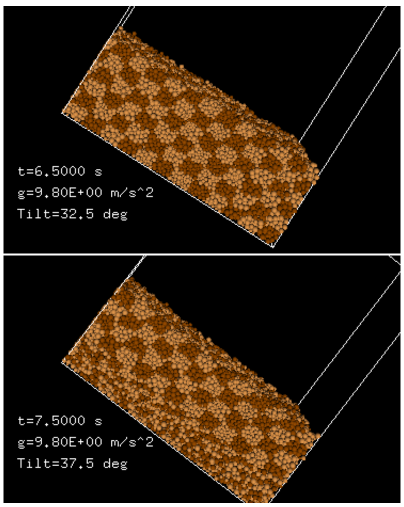

Figure 1: Granular bed in a box with periodic boundaries. $\mu_k = 0.5$ and $\mu_r = 0$. The images show the particles when they have started to flow and a second after, when the flow has been stablished.

Fig. 1 shows images of one of our simulations ($\mu_k = 0.5$, $\mu_r = 0$). Fig. 1 (top) shows the granular bed as the particles have just passed the static angle of repose and start to flow. Fig. 1 (bottom) shows the bed with a fully developed granular flow. The images show the motion of the particles and the expected velocity gradient. Fig .2 shows the variation of the static angle of repose for different values of μ_k and μ_r. Each line represents a value of μ_r and we have plotted only their upper bound. As explained above, the inclination was carried out in steps, so this is the precision of the measurements. These results are similar to those found by [9] and reconfirm that the code follows standard implementation of the SSDEM.

The problem of a vertical column that collapses under the effects of gravity was also studied. Initial runs indicate that the resulting slope of the granular bed ranges from 0 to $\approx 18°$ for $\mu_r = 0$ and μ_k in the above mentioned range. These results agree with the findings of [8].

Given that DEM codes are used to study the interaction of exploration pods, projectiles and landing pads with the regolith-covered surface of asteroids, it is also

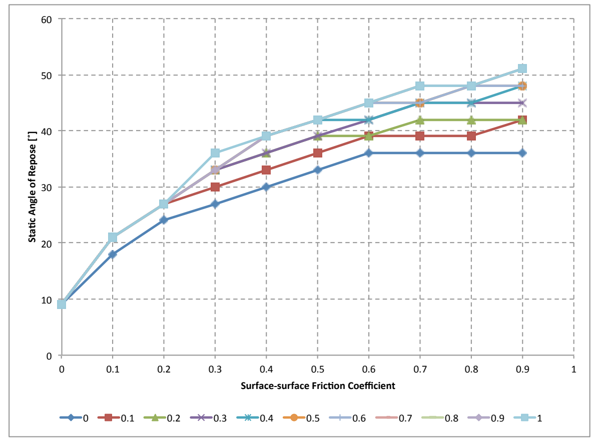

Figure 2: Variation of the static angle of repose of the granular bed with the coefficient of surface-surface friction (μ_k). Each line corresponds to a single value of the coefficient of rolling friction (μ_r).

important to analyze low speed impacts and penetrometry. The results from these simulations, as well as from others carried out to to study the dynamic angle of repose that our code produces will be analyzed in detail during the conference.

Conclusions: This paper studies the influence of the coefficients of dynamic and rolling friction in a simulation code that implements a SSDEM, the same that has been used to investigate the dynamics of granular asteroids and their surfaces. We find that the code works as expected and in agreement with previous simulation efforts. Further exploration of the parameter space, as well as other geometries and tests, regarding penetrometry at low speed impacts are planed.

References: [1] P. Cundall (1971) in *Proceedings of the International Symposium on Rock Mechanics* vol. 1 129–136 -, Nancy. [2] S. Luding (1998) *Physics of Dry Granular Media* NATO ASI Series, Dordrecht Kluwer Academic Publishers. [3] P. Sánchez, et al. (2011) *The Astrophysical Journal* 727(2):120. [4] M. P. Allen, et al. (1989) *Computer Simulation of Liquids* Oxford science publications Oxford University Press, USA, New York ISBN 0198556454. [5] L. E. Silbert, et al. (2001) *Phys Rev E* 64(5):051302 doi. [6] J. Ai, et al. (2011) *Powder Technology* 206(3):269 ISSN 0032-5910 doi. [7] P. Richard, et al. (2008) *Physical Review Letters* 101(24):248002 doi. [8] K. A. Holsapple (2013) *Planetary and Space Science* 8283:11 ISSN 0032-0633 doi. [9] H. Maleki, et al. (2008) *Journal of Statistical Mechanics: Theory and Experiment* 2008(04):P04026.