A METHOD TO GENERATE PERIODIC ORBITS NEAR DUMBBELL-SHAPED ASTEROIDS WITH APPLICATION TO 216 KLEOPATRA.
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Introduction: Natural periodic orbits around irregular shaped asteroids are significant for the understanding of the dynamical behaviors in their gravitational fields. Recognizing their different sizes, shapes and compositions due to the large number of small celestial bodies, the dumbbell-shaped asteroids (including comets) are the main focus of this study as representatives. The equations for a test particle are usually expressed in the body-fixed frame for a uniformly rotating asteroid with

$$\ddot{\boldsymbol{r}} + 2\boldsymbol{\omega}\times\dot{\boldsymbol{r}} + \boldsymbol{\omega}\times\left(\boldsymbol{\omega}\times\boldsymbol{r}\right) = -\nabla U \tag{1}$$

where $\boldsymbol{r}$ is the body-fixed position vector from the bary-center of the asteroid to the particle, $\boldsymbol{\omega}$ is the rotating angular velocity vector with respect to the inertial reference frame, and U is the gravitational potential of the irregular shaped asteroid.

In this paper, two models will be adopted to study large-scale periodic orbits over natural elongated bodies, i.e., the rotating mass dipole [1] and the relatively precise polyhedral model [2]. The exterior gravitational potential of an asteroid or comet is first approximated by the rotating mass dipole [3]. The dipole model is composed of two point masses connected with a massless rod in a constant length d. The phantom rod can provide compressive or tensile forces resulting in the two point masses in a fixed relative position. A body-fixed frame can be defined with its origin at the mass center of the dipole system. The polyhedral method is also briefly introduced as it is only used in the final calculation of the potential of minor bodies. It can express the exterior gravitational potential and accelerations in a closed form. An irregular shaped minor body should be first discretized into groups of faces and edges based on its exterior shape which was usually obtained with radar observations or vicinal explorations [4].

In this paper, large-scale periodic orbits around the dipole model will be searched as the first step which are adopted as the initial guess for the solutions around realistic dumbbell shaped asteroids. Local iterations are then performed to obtain the exact values of periodic solutions around the asteroid in the polyhedral model. The feasibility of the above method can be conjectured from two main reasons. The first is that both of the two models have the same form of dynamic equations and correspond to the Hamiltonian system resulting in the first integral of Jacobi constant. The second comes from the fact that topological structures around exterior equilibrium points of the two systems for sample asteroids coincide with each other according to Zeng et al. [3].

Additionally, such a method is only to provide an attempt to solve the difficult problem in an easy way as the circular restricted three body problem (CRTBP) was used to penetrate the three body problem [5]. Particularly, the dynamic equations between the CRTBP and the dipole model hold the same mathematical form but slightly different potentials with only one different parameter. Thus, when we try to search for periodic orbits around the dipole model, previous methods and results about the CRTBP can be employed as references for the current problem.

In this study, the asteroid 216 Kleopatra is adopted as the representative for dumbbell-shaped asteroids. Figure 1 shows the distribution of outer equilibrium points as well as zero-velocity curves in the equatorial plane. Note that the gravitational potential distribution inside the body's surface cannot be approximate by the dipole model. Here, only the four exterior equilibria are given by neglecting the inner equilibrium point (or points). Thus, the name of these equilibria pinpointed with E_1 to E_4 is different from the traditional Lagrange points of the circular restricted three body problem (CRTBP).

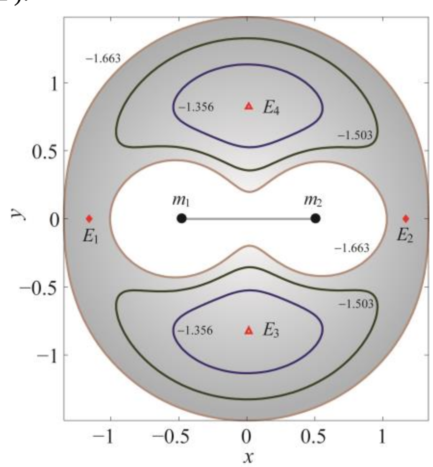

Figure 1. Equilibria of 216 Kleopatra generated by the dipole in dimensionless units

Practical Procedures: Although there are symmetrical planes for the dipole model, such planes do not exist due to the irregular shape of natural elongated bodies. In order to search for three dimensional periodic orbits around realistic minor bodies, the symmetrical characteristics of the dipole system will not be used for reduction. All steps for searching periodic orbits can be listed as follows:

(1) Give the values of two direction angles ψ and φ to fully determine the section of surface shown in Fig. 2 [6]. In order to search periodic solutions as many as possible, the angle θ of the initial position vector in the section plane is varied in relatively small steps, such as 0.01π to produce 200 nodes of the interval $[-\pi, \pi)$. Generate a constant value v_0 for the magnitude of the initial velocity as well as r_0 (whose least value should be guaranteed outside the central body's surface).

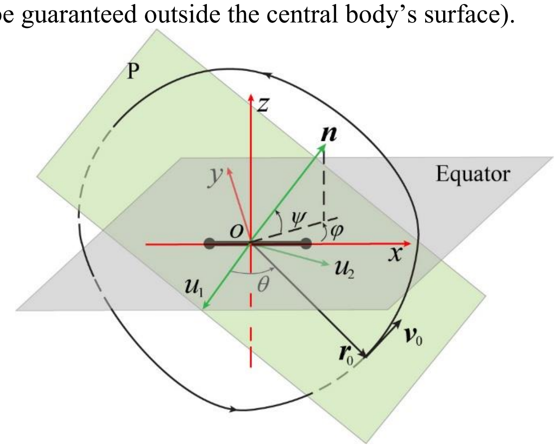

Figure 2. Skematic map of a periodic orbit over the dipole model and the section surface

(2) After obtaining the initial guess of state vectors $\boldsymbol{r}_0$ and $\boldsymbol{v}_0$ with the above five parameters, first check the system energy whose value must be negative to avoid initial hyperbolic trajectories. As for the numerical integration of the trajectory, the calculating time is set to be long enough, such as 60π, to check the propagation of the test particle. If its final position is still in the near realm of the central body, then check the cut-off condition that the trajectory returns to the section surface along its positive normal direction (including multiple crossing).

(3) Check the residual between the initial guess $\boldsymbol{w}_0$ and the final state $\boldsymbol{w}_f$ satisfying the cut-off condition. If the residual $\Delta s = \| \boldsymbol{w}_f - \boldsymbol{w}_0 \|$ is less than the predetermined tolerance (for instance 10^{-1} to keep all possible solutions), a modified Powell's hybrid algorithm is used to adjust the initial guess to further decrease the residual [7].

(4) Each output after the above step is considered to be a potential periodic solution. Local iterations based on the differential corrections by using the state transition matrix are implemented to improve the accuracy of the initial state variables of the required periodic orbit. Therefore, the linear stability of the orbit around the dipole model can be obtained without additional calculations.

(5) With the above four steps, the required periodic orbits around the dipole model have been obtained. Taking $[\boldsymbol{w}_0, T_0]^T$ as the initial values over a real minor body, recalculate the orbit by using its corresponding polyhedral model.

Sample orbits: The family of periodic orbits are single lobe orbits with a small loop crossing the plane *yoz*. The vertical motion near the body's surface will be enlarged along with the decreasing of the orbital period. All these orbits are unstable where a four-dimensional unstable sub-manifold of each orbit is maintained with $1, 1, e^{\pm\alpha j}$ ($\alpha_j \in R$, $\alpha_j > 0$; $j = 1, 2$).

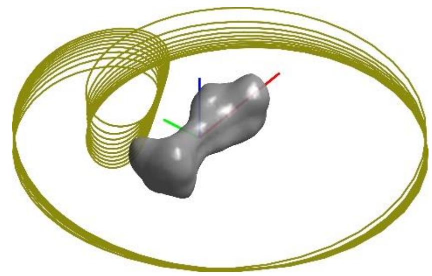

Figure 3. Single lobe orbits with a vertical mode around 216 Kleopatra

Besides the above families of periodic orbits successfully obtained from the initial guesses, an unexpected case was also encountered during the local iterations. More detailed discussions regarding the calculations of these periodic orbits as well as other obtained families will be given in the coference presentation.

Conclusion: A practical method is proposed to efficiently obtain periodic orbits over dumbell-shaped asteroids. The required initial six state variables and the orbital period are first obtained around the dipole model, which is used to approximate the exterior potential distribution of an elongated asteroid or comet. Taking those seven parameters as an initial guess, local iterations are then performed to identify the real orbits around the minor body. Sample orbits around Kleopatra are given to validate the effectiveness of the method.

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