LUNAR TIDAL RECESSION. James G. Williams, Dale H. Boggs, and J. Todd Ratcliff, Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA, 91109, USA (e-mail James.G.Williams@jpl.nasa.gov).

Introduction: The orbit of the Moon is our earliest example of tidal evolution. It is a prototype for tidal evolution in the solar system that includes the Earth with its oceans, rocky planets and satellites, icy satellites and Kuiper belt objects, and the four outer planets with their massive atmospheres. Tidal evolution also affects many exoplanets. In this abstract we summarize knowledge of the secular rates of change of the lunar orbital elements and the Earth's rotation.

The majority of the terrestrial tidal dissipation takes place in the oceans, but some takes place in the Earth's rocky mantle [1]. Also, some dissipation takes place in the Moon [2,3].

Analysis of Lunar Laser Ranging (LLR) data provides the results given here [4]. Ranges are measured by firing a laser pulse from an observatory on the Earth that strikes retroreflectors on the Moon and bounces back to the Earth. The Moon is receding from the Earth by 38 mm/yr due to tides and the eccentricity is increasing by 1.5×10^{-11} /yr. Inclination change is very small. As a result of the loss of energy and angular momentum, the Earth's spin is decreasing. The length of day is predicted to increase by 2.39 msec each century from tidal dissipation.

Tidal Acceleration: At ±1 decade from a reference time, tidal acceleration alters the lunar distance by ±13 m and this monthly oscillation grows quadratically with time. To fit LLR data with centimeter or better accuracy, we need a tidal model that represents the long-term forces from tidal dissipation to ≤0.1%. Although ocean tides are more complex than solid body tides, for long-term changes we concentrate on a selection of degree-2 tidal components that are proportional to the periodic components of the tide-raising potential. Love numbers quantify the distortion of the body and its gravity field. The LLR model for integration of the orbit has Love numbers and time delays for zonal, diurnal, and semidiurnal tidal bands: 3 Love numbers (k_{20}, k_{21}, k_{22}) for the three frequency bands, one zonal time delay (τ_0), and two time delays each for diurnal and semidiurnal tides [4,5]. The latter two pairs of time delays allow for a frequency dependence of the dissipation associated phase shift by assigning separate delays to rotation (τ_{R1}, τ_{R2}) and orbit (τ_1, τ_2).

Geophysical Model: Love numbers describe the tidal distortion in response to the tide-raising potential. Dissipation introduces phase shifts that can be accommodated by making the Love numbers complex. We generated complex Love numbers at a sequence of tidal periods for an Earth model with oceans [6]. The tidal response of Earth without oceans is important for tide height; our calculation allows for a near diurnal resonance due to the flattening of the core-mantle boundary. The ocean model dominates the phase shift; we use FES2004 [7] and its tidal potential [8].

The geophysical model is used in two ways: to predict the secular tidal rates of lunar orbit and earth rotation, and to aid the LLR solutions. For the latter, the geophysical model provides the three Love numbers for the three tidal bands plus the time delay for the zonal tides. It also provides one of the two time delays for diurnal tides and it can also provide one semidiurnal time delay. One diurnal and one or two semidiurnal delays are solution parameters.

The model for tides on the Moon is described in [3,5]. The model in the integrator has one Love number and one time delay. The lunar Love number k_2 comes from GRAIL analysis [9,10,11]. Also, the fitting routine solves for tidal displacement Love number h_2 and periodic tidal terms in the lunar orientation.

LLR Solutions: The Lunar Laser Ranging (LLR) data, consisting of 20,218 ranges extending from March 1970 to September 2015, are analyzed. The accuracy has improved with time. On the Earth, McDonald Observatory, Texas, Observatoire de la Côte d'Azur, France, Haleakala Observatory, Hawaii, Apache Point Observatory, New Mexico, and Matera, Italy have provided data sets extending over years. Ranges to 5 retroreflectors at different lunar sites provide information on the Moon's orientation vs. time. LLR data are available from the International Laser Ranging Service archive [12].

The analysis of LLR data includes solving for two or three terrestrial time delays, a lunar tidal time delay, and a parameter for dissipation at the lunar core-mantle boundary. Standard solution parameters for station positions, Earth orientation, retroreflector positions, and lunar initial orientation and orbit are also included.

For the orbit, the most important diurnal tides are the O1 and Q1 tides and the most important semidiurnal tides are M2 and N2. The M2 tide has a period equal to half of the mean time between meridian crossings (argument $2\theta–2L$, where θ is the rotation angle of the Earth and L is the lunar mean longitude). The N2 tide has a slightly different period since it also depends on lunar mean anomaly ($2\theta–2L–l$). The diurnal tides depend on latitude and longitude; the orbit is most affected by the O1 ($\theta–2L$) and Q1 ($\theta–2L–l$) tides.

We consider several LLR solutions. 1) First, we use the geophysical model for τ_1, and τ_2, which establishes the phase shift from O1 to Q1 and M2 to N2, while solving for one diurnal and one semidiurnal time delay (τ_{R1}, τ_{R2}). With that choice we also improve the fit by solving for an extra de/dt rate that we have found

in the past [13]. 2) Another solution only uses the geophysical information to establish the (τ_1) link between O1 and Q1 while solving for one diurnal time delay (τ_{R1}) and one semidiurnal time delay (τ_{R2}) while adjusting τ_2. The latter solution did not require any extra de/dt rate; the extra de/dt rate was replaced by a 17% larger phase shift for the N2 tide.

Secular Rates: We wish to convert the Love numbers and time delays into average rates for semimajor axis a, mean motion n, eccentricity e, and inclination i. In the past [13,14] we have used an analytical theory based on a precessing ellipse. Recently we have developed a theory based on a Fourier expansion of tides. The new expressions allow for periodic solar perturbations of the orbit that were previously lacking.

The conversion of tidal parameters from the second LLR solution gives da/dt = 38.30±0.09 mm/yr, dn/dt = –25.97±0.06 "/cent^2, de/dt = (1.50±0.10)×10^{-11} /yr, and di/dt = –0.5±0.1 μas/yr. Consequently, the perigee is increasing by 30.4 mm/yr and the apogee is increasing by 46.2 mm/yr. For da/dt and dn/dt, the M2 tide causes 81% of those total rates, 12% is from the O1 tide, and 7% is due to the N2 tide. Including smaller contributions, semidiurnal tides cause 88% of the rates and diurnal tides cause 13%, while Moon tides at 1% and zonal tides at 0.6% act with opposite signs. For total de/dt, the N2 tide causes 138% of the total, the Q1 tide contributes 14%, the M2 tide opposes that with –7%, and tides on the Moon reduce the total with –38%. Tides on the Moon cause much of the small inclination decrease.

When the geophysical model is converted to secular rates for the orbit, da/dt and dn/dt are predicted to be 0.7±0.3% larger than the LLR result. Most of that difference is from a 7±3% difference in the diurnal contribution. The eccentricity rate from the Earth fares much worse; the geophysical model is 14% lower than the LLR analysis indicates with the N2 tide suspected to be the major difference. The geophysical model also predicts the tidal slowing of the Earth's rotation rate. With a small correction for the 0.7% difference of da/dt from LLR, we predict that dissipation from Moon and Sun caused tides increases the length of day by 2.395 msec each century.

Evolution: Tidal dissipation in the Earth's oceans varies by a large amount as the continents shift from plate motion. Calculations of paleotides [15] suggest that the M2 tidal dissipation can vary by large amounts. The present configuration of continents and oceans is very effective at tidal dissipation. At times of low terrestrial tidal dissipation, tides in the Moon might cause the total eccentricity rate to be negative.

Summary: Tides in the mantles of the Earth and Moon should change very slowly due to cooling, but tides in the oceans are complicated and will change due to plate motion. Analysis of Lunar Laser Ranging (LLR) data provides a refined determination of 38.3 mm/yr for the rate of recession of the Moon's semimajor axis due to tidal dissipation. Tidal dissipation also affects eccentricity rate; the LLR analysis finds (1.50±0.10)×10^{-11} /yr. The perigee increases 30.4 mm/yr and the apogee increases 46.2 mm/yr. Over long times, tidal dissipation slows the Earth's rotation.

A geophysical model predicts the semimajor axis rate with an error of ≤1%. The diurnal dissipation accounts for most of that difference. The model value of the eccentricity rate, 15% low, would agree if the phase shift of the N2 tide were increased.

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