Crater Equilibrium as an Anomalous Diffusion Process

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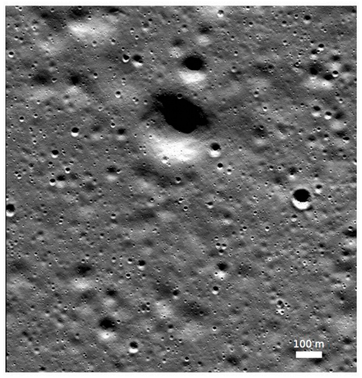
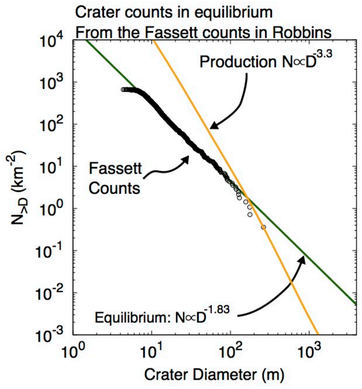
So, you think you understand crater equilibrium?

Portion of LRO NM146959973LAC image used in Robbins et al. (2014)



Crater counts in equilibrium
From the Fassett counts in Robbins



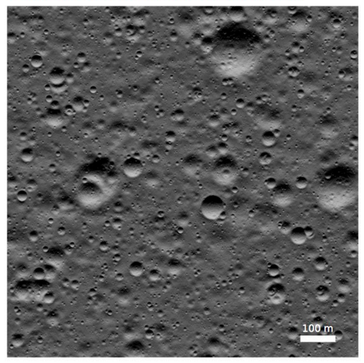
Obervations of heavily cratered terrains define an "empirical equilibrium" level as a cumulative power law with a slope of -1.83 (Hartmann 1984)

Very heavily cratered terrains appear to reach an equilibrium in the total cumulative number of countable craters per unit area.

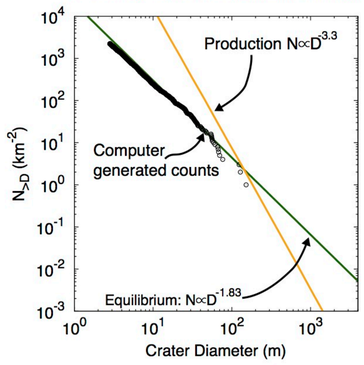
But what is the process that determines the observed crater count equilibrium level?

Here we use the Cratered Terrain Evolution Model (Richardson 2009, Minton, Richardson, and Fassett 2015) to investigate crater count equilibrium.

Shaded digital elevation model generated by the CTEM, the Cratered Terrain Evolution Model



Crater counts generated by CTEM, in which we successfully reproduce equilibrium cratering.

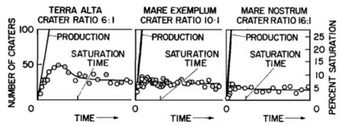


We found that the observed equilibrium level is a consequence of crater degradation due to downslope diffusion. But classical diffusion is inadequate to capture the process. Downslope movement is instead an anomalous diffusion process, similar to lateral transport as shown by Li & Mustard (2000).

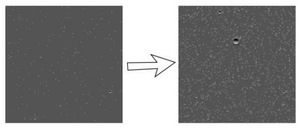
History of equilibrium cratering modeling

Experimental and Observational

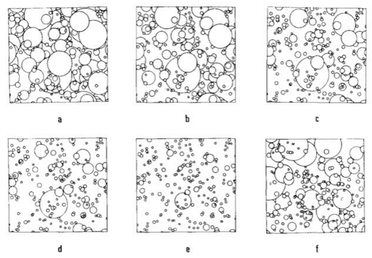
Gault (1970) showed that natural and experimental cratered surfaces (see left figure) reach a maximum crater density of about 5-10% of "geometrical saturation."



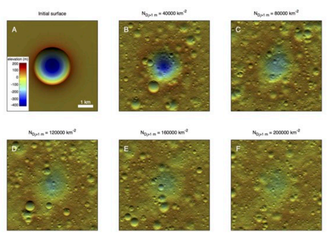
Numerical



Monte Carlo terrain evolution codes like CTEM are the primary way that cratered surfaces are modeled numerically. Monte Carlo codes can be broadly categorized as either circle-based or topography-based.



Circle-based codes (above from Woronow 1978), model craters as circles. Circles represent crater rims. Crater erasure is parameterized by a factor that determines whether a crater of a given size can erase the rim of another crater. See also Chapman & McKinnon (1986); Marchi et al. (2012).



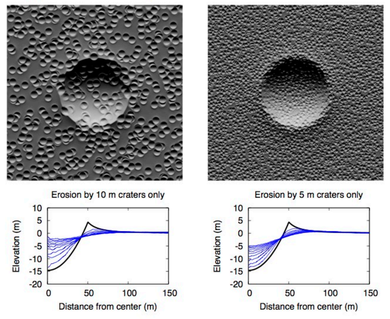
Real craters on heavily cratered terrains are erased by diffusive erosion (i.e. Fassett and Thomson 2014), not by having their rims overprinted. Topography codes like CTEM (see also Gaskell & Hartmann 1997), which represent the cratered terrain as a three-dimensional digital elevation model, can capture this mechanism in a more natural way than circle-based codes.

How we modeled equilibrium cratering in CTEM

This illustration from Ross (1968) shows how small craters erode larger craters.

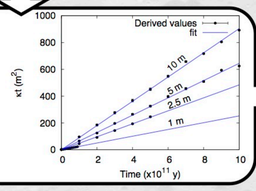
To model this in CTEM we need to know what is the erosive "power" of each crater.

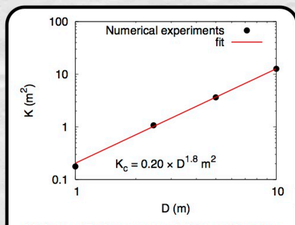
We accomplished this with a series of numerical experiments in which we generated a 100 m test crater and bombarded it with a size-distribution containing a single size of crater, from 1 - 10 m.



Shown above are the results of two of our simulations. Over time, the single-size craters erode the larger crater, softening its profile.

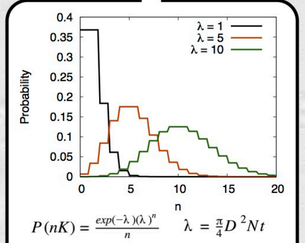
Profiles of the 100 m test crater can be fit the profile of the same crater being eroded due to diffusion, using a classical diffusion model. Each crater size gives a different diffusion constant.





$K_c = 0.20 \times D^{1.8}\ m^2$

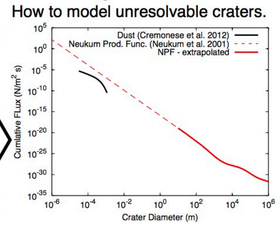
Using a Poisson model to estimate the number of overlapping craters over the test area over time, we solved for the intrinsic per-crater diffusive power. This tells us how much each crater contributes to the diffusivity of the system.



$$P(nK) = \frac{exp(-\lambda)(\lambda)^n}{n} \qquad \lambda = \frac{\pi}{4} D^2 N t$$

To get what we need, which is a model of the intrinsic per-crater diffusive power, Kc, we must turn to Poisson statistics. This allows us estimate the amount of times a point on the surface is overlapped (n) by a crater of diameter D and diffusive power K_c, given an apparent surface-wide diffusion κt.

With an estimate of the intrinsic per-crater diffusive power, we can tackle one of the big issues with CTEM: How to model unresolvable craters.

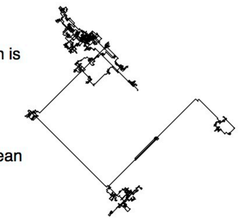


We adopt the Neukum Production Function (NPF) as our cratering model. To estimate the effects of unresolvable craters, we extrapolate the cumulative slope of the NPF at ~10 m down to micron sized dust. Based on LDEF experiment dust measurements, this likely over-predicts the primary production population.

Classical Diffusion

Unlike the raindrops that erode desert landscapes on Earth, impacts do not have an upper bound on size. Therefore the random walk in topography associated with impact erosion is best modeled as anomalous diffusion, rather than classical diffusion.

In anomalous diffusion, the mean square displacement of the system has a power-law size dependence. In the case of cratering, the more time passes, the more likely a given piece of surface will be hit by a bigger impactor.

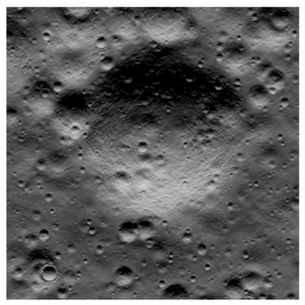


Anomalous Diffusion

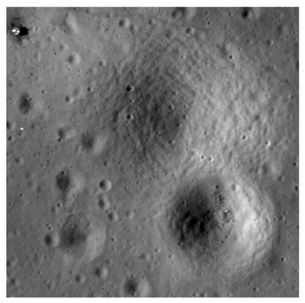
Unresolvable craters in (smaller than the size of a grid cell) are modeled using anomalous diffusion.

$$\frac{\partial z}{\partial t} = \frac{\partial}{\partial x}\left(K_d \frac{\partial z}{\partial y}\right) + \frac{\partial}{\partial y}\left(K_d \frac{\partial z}{\partial y}\right)$$

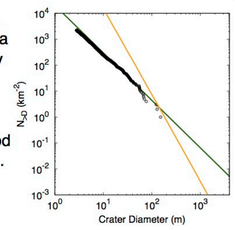
At each time step, a diffusion parameter Kd is drawn from a Poisson distribution that couples our per-crater diffusion model to the production function. This is done for each grid cell, and so we use a finite difference scheme that allows for spatially varying diffusivity.

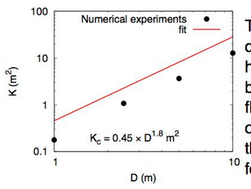


When we implemented our anomalous diffusion model for sub-pixel craters, we got an unexpected result: The slopes of craters started to develop a texture reminscent of the "elephant hide texture" that is often associated with lunar slopes. On the left is a portion of a 1 m/px simulation from CTEM, and on the right is an LROC image of the Apollo 14 landing site, showing a similar texture.



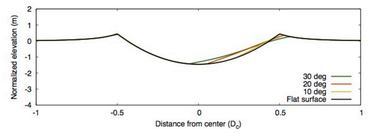
After some trial and error, we found that adding extra intrinsic diffusivity (slightly more than doubling it) to both resolvable and unresolvable craters allowed us to reach a good fit to empirical equilibrium.



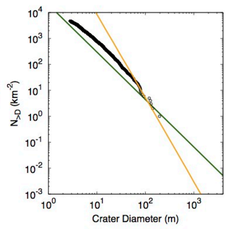


$K_c = 0.45 \times D^{1.8}\ m^2$

The reason for this extra diffusion is unknown. We hypothesize that it may be related to acoustic fluidization, or some other dynamic process that enhances collapse for craters on slopes.



Even with our well-calibrated (and likely over-estimated) anomalous diffusion model for small, unresolvable craters in place, we still generated higher than equilibrium-level numbers of craters!



We already have a calibrated ejecta blanket burial model in CTEM (Minton, Richardson, and Fassett 2015). CTEM can model how old craters are erased beyond the rims of new craters in a self-consistent way.