Modeling the Evolution of the Sphere-Restricted Full Three-Body Problem. Travis S. J. Gabriel[1] and Daniel J. Scheeres[1], [1]Department of Aerospace Engineering, University of Colorado Boulder, CO 80309, Travis.Gabriel@colorado.edu, Scheeres@colorado.edu.

Introduction: In Scheeres (2014) the Sphere-Restricted Full Three-Body Problem (RF3BP) is outlined as a particular aspect of a problem set regarding full-body dynamical systems [1]. Full-body problems host particles of finite density, disallowing them to come infinitely close to one another as in classical dynamics. By allowing for the dissipation of energy in the system, a discrete set of equilibrium states can be achieved - all of which are listed in Figure 1. By applying friction and restitution, potential relative equilibria will include resting states where relative motion between objects is zero. The solution space has been described in terms of stable and unstable states assuming spheres of uniform density and mass [2].

Using perfectly-rigid dynamics, we simulate systems with a given angular momentum in a purpose-written C-based code. This investigation explores various restitution and friction coefficients for given initial angular momentum. We aim to describe the relative mechanics and dynamics in rubble-pile bodies where relative forces are not great enough to compromise the rigidity of the constituents. All output is analyzed in terms of normalized units, with the gravitational parameter and the minimum distance between the equal spheres as unity.

Computation: A C-based Discrete Element Method (DEM) code has been developed for the study of full-body dynamical systems. The code resolves perfectly rigid-body interaction between spheres with surface friction and of various densities and masses. Impact mechanics are dictated by restitution and friction. Such a formulation is subject to inelastic collapse in the case of a less-than-unity coefficient of restitution, where an infinite number of impacts occur in a finite time, i.e. dissipation results in relative velocities tending to zero, requiring the timestep to do the same in order to resolve the interaction. We implement a variable coefficient of restitution for cases of negligible relative velocity to mitigate this issue – applying a unity restitution condition. In a strict sense, this generates resting solutions that should be described as 'pseudo-resting' since persistent contact is not explicitly handled. Numerically resolving persistent contact requires the employment of soft-body mechanics, whose contact interaction is usually modeled by spring dampeners; however, this method also encounters inelastic collapse in the limit of perfectly rigid objects, an assumption that is intrinsic to the full-body problem formulation in Scheeres (2012).

Euler-Cromer integration is compared to that of Velocity-Verlet integration for the case of the RF3BP. We confirm that energy and angular momentum are conserved to within the limits commonly accepted in the computational granular physics community.

Initial Conditions. Each of the three equal spheres is placed in either an Euler (aligned) or Lagrangian (equilateral triangle) non-resting configuration, with some global circular velocity about the barycenter. To eliminate the possibility of an early dense interaction (two simultaneous contacts) the initial configuration is negligibly and non-uniformly perturbed.

Results: We find that the mechanism of dissipation can have a significant effect on the final equilibrium configuration of the RF3BP and the time in which this state is achieved. It is reasonable to suspect that this result extends to the Restricted Full N-Body Problem. The existence of two possible equilibrium configurations for a given angular momentum in the RF3BP is found analytically in previous work [1]. The DEM simulation environment developed herein affords the opportunity to examine the evolution of a full-body system in the context of a strictly defined dynamical problem near areas of mixed solutions. This precursor study is part of a larger fundamental effort to outline the dynamical pathways that restricted full-body configurations will trace in phase space given various initial conditions and models of dissipation. Additionally, we aim to use these results as an analogy for rubble-pile solar system bodies in an attempt to describe their past, present, and future dynamical-mechanical states.

Equilibria in the Spherical, Full 3 Body Problem with Equal Size

Configuration	Name	Energetic Stability	Conditions
	Lagrange Resting	Stable	
	Euler Resting	Stable	For high enough H
	Aligned	Stable	Outer Solution
	Lagrange	Unstable	
	Euler	Unstable	
	Euler Resting	Unstable	For low H
	Aligned	Unstable	Inner Solution
	Transverse	Unstable	
	Transitional	Unstable	

Figure 1: Table of equilibria for the RF3BP [2].

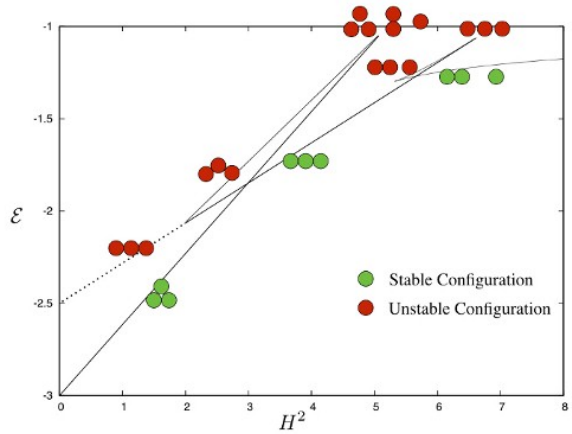

Figure 2: Relative equilibria chart of the RF3BP with normalized energy and angular momentum squared in the vertical and horizontal axis, respectively [2].

References: [1] Scheeres (2012) Celestial Mechanics and Dynamical Astronomy, 113, 3, 291-320. [2] Scheeres (2014) Complex Planetary Systems, Proceedings IAU Symposium, 310.