Scaling Rule Between Cohesive Forces and The Size of a Self-Gravitating Aggregate

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This paper presents a series of physics models implemented into a Soft-Sphere Discrete Element Method (SSDEM) code to study the evolution of self-gravitating granular aggregates that are spun to disruption. We use these aggregates as a proxy for understanding "rubble-pile" asteroids that are spun up to high rotation rates under the influence of the YORP effect. The scaling properties of this model are explored. When cohesion is added, the system is no longer size and density independent, providing a means for simulating asteroids of different sizes through changing a single non-dimensional cohesion parameter.

Introduction:

A recent achievement in understanding the strength of asteroids was the development of a theory for how cohesive forces could arise in rubble pile asteroids [1]. In this model dry cohesive forces (e.g., van der Waals forces) between fine grains of regolith can form a matrix within which larger boulders can be emplaced. The limiting strength of such loosely packed "cements" can be very weak, on the order of 25-100 Pa, yet in the micro-gravity environment found on asteroids this can dramatically change the failure behavior of rubble piles [1].

The current paper incorporates this cohesion model into an SSDEM simulation code to explore the effect of cohesion on the failure spin rates for rubble pile bodies. In addition, a theoretical analysis of the problem is made that indicates how a cohesive rubble pile simulation can be scaled to describe a range of different rubble pile sizes without having to adjust the total number of particles in the aggregate. The significance of this result is in that it opens the door for more efficient simulations and analyses of size dependence on failure modes in rubble pile asteroids.

Theoretical Analysis:

The interactions between grains in a rubble pile asteroid consist of gravitational attractions between grains, cohesive forces between grains in contact, and the resulting normal and tangential contact forces. The equation of motion for a single grain of radius R_i expressed in a body-fixed, rotating frame can then be expressed as:

$$\begin{aligned} m_i\left[\ddot{\boldsymbol{r}}_i + 2\boldsymbol{\omega}\times\dot{\boldsymbol{r}}_i + \dot{\boldsymbol{\omega}}\times\boldsymbol{r}_i + \boldsymbol{\omega}\times\boldsymbol{\omega}\times\boldsymbol{r}_i\right] = \\ m_i\boldsymbol{g}_i + \boldsymbol{N}_i + \boldsymbol{T}_i + \boldsymbol{C}_i \end{aligned} \tag{1}$$

where m_i is the mass of the grain, $\boldsymbol{r}_i$ is the position of the grain in the body frame, all time derivatives are taken in the body-fixed, rotating frame, $\boldsymbol{\omega}$ is the instantaneous angular velocity of the body, $\boldsymbol{g}_i$ is the gravitational acceleration acting on the grain, $\boldsymbol{N}_i$ is normal force acting on the grain, $\boldsymbol{T}_i$ is the tangential frictional force acting on it, and $\boldsymbol{C}_i$ is the cohesive force between boulders connected by the matrix. In our model the magnitude of the cohesive force between two boulders is expressed as the sum of multiple grains in proximity

$$\boldsymbol{C}_i = \sum_{j\in I_i}\boldsymbol{C}_{ij} \tag{2}$$

where $j \in I_i$ denotes all of the boulders in contact with boulder i.

The gravitational acceleration acting on our grain will be of the form

$$\boldsymbol{g}_i = -\mathcal{G}\sum_{j=1,j\neq i}^{N}\frac{m_j}{r_{ij}^3}\boldsymbol{r}_{ji} \tag{3}$$

with the summation occuring over all boulders.

If the rubble pile is uniformly rotating and the grain is stationary in the body frame then we have a balance between gravitational, cohesive, centripetal and restoring forces. Thus, when at rest each grain is subject to the force balance

$$\boldsymbol{N}_i + \boldsymbol{T}_i = m_i\boldsymbol{\omega}\times\boldsymbol{\omega}\times\boldsymbol{r}_i - m_i\boldsymbol{g}_i - \boldsymbol{C}_i \tag{4}$$

We note that the gravitational and centripetal accelerations can be made scale independent, but that the cohesive force will not be scale independent in general. The normal and tangential forces are just driven by the other forces, and thus will scale relative to the forces that hold the grains together.

It is possible to normalize the equations of motion to expose the fundamental scaling parameters of interest by introducing a mass scaling (total mass of the aggregate), a length scaling (radius of a grain), and a time scaling (orbit period at the surface of the aggregate).

If there is no cohesion present, then the contact forces are completely driven by the scale invariant gravitational and rotational accelerations. Thus, without cohesion the behavior of a rubble pile asteroid is ideally independent of size. However, the presence of cohesive forces between larger grains through our "matrix" model of regolith breaks this size and density scale independence. This introduces a non-dimensional parameter (c) that enables us to evaluate different sized rubble pile asteroids through adjusting a single scaling parameter that defines the relative strength of cohesion.

Thus, assuming a fixed boulder size R_i and matrix strength σ_{YY} (or a fixed ratio between them), the effect of changing the total rubble-pile asteroid size, R, changes the dimensionless cohesion as $1/R^3$. As an asteroid becomes larger, it is possible to show that the effective cohesion becomes small, while as an asteroid becomes smaller the cohesion grows. Specifically, if all other cohesion quantities are kept fixed except for the total body size R, then:

$$cR^3 = c'R'^3 \quad (5)$$

where c is the dimensionless cohesion.

Simulation:

The simulation program that is used for this research applies a Soft-Sphere Discrete Element Method [2, 3, 4, 5] to simulate a self-gravitating granular aggregate. The particles, modelled as spheres that follow a predetermined size distribution, interact through a soft-repulsive potential when in contact. This method considers that two particles are in contact when they overlap. When this happens, normal and tangential contact forces are calculated [6]. The former is modelled by a linear spring-dashpot system and is always repulsive, keeping the particles apart; the latter is also modelled with a linear spring that satisfies the local Coulomb yield criterion.

In addition, we have also implemented rolling friction [8]. For this, a winding spring provides a torque to particles in contact. In form, it is very similar to surface friction, but is related to the relative angular displacement. This type of friction allows us to obtain aggregates with angles of friction of up to $\sim$35°, typical of cohesionless aggregates on Earth, though angles of $\sim$40° are not rare.

The particles (initially cohesionless and frictionless) are left to coalesce in the desired shape so forming very homogeneous internal structures. The aggregates are spun up in small, discrete increments around their CM and the disruption process is observed.

Fig. 1 shows the scaled results of the simulations that were carried out with perfectly spherical aggregates, $\approx$142 m in size.

In spite of this, the results obtained here do agree with our theoretical predictions for the spin rate limit of granular aggregates which cohesive strength originates in molecular Van der Waals forces existent among the fine regolith that would form a matrix.

Fig. 2 shows the disruption of some of the simulated aggregates that started as spheres. The different disruption modes are a direct result of the increased strength of the cohesive bonds. However, given the scaling relationship between cohesive forces and size, it is possible to see these simulations as of aggregates with the same cohesive strength but different size.

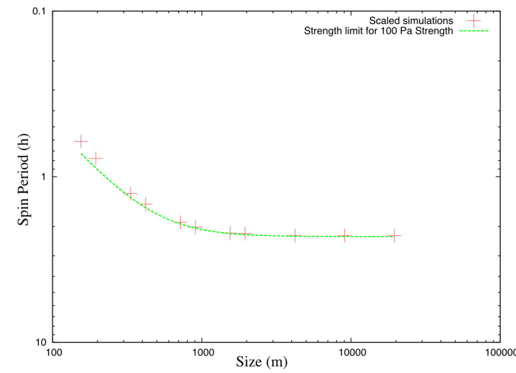

Figure 1: Spin rate versus size data, theoretical strength curve, and the simulation results.

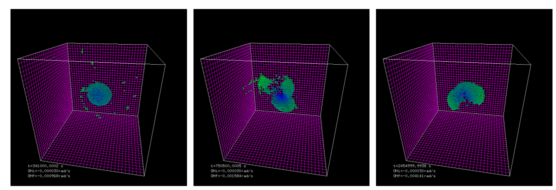

Figure 2: Disruption of the simulated aggregates. Size ratios: 27.2:2.2:1

Conclusions: This analysis shows that asteroids of similar size would behave differently if their cohesive strength is different (which is exactly what we have done in simulations). However, more importantly, asteroids with different size could behave differently even if their make-up is the same. An additional benefit of this scaling relationship is that it allows us to translate the results obtained in small granular systems to larger ones without the increased computational cost due to the incremented number of particles or the unrealistically large particle size. This and other implications will be analyzed at the conference.

References: [1] P. Sánchez, et al. (2014) *Meteoritics & Planetary Science* 49(5):788 ISSN 1945-5100 doi. [2] P. Sánchez, et al. (2011) *The Astrophysical Journal* 727(2):120. [3] P. Cundall (1971) in *Proceedings of the International Symposium on Rock Mechanics* vol. 1 129–136 -, Nancy. [4] P. A. Cundall, et al. (1992) *Engineering Computations* 9(2):101 doi. [5] M. P. Allen, et al. (1989) *Computer Simulation of Liquids* Oxford science publications Oxford University Press, USA, New York ISBN 0198556454. [6] H. Herrmann, et al. (1998) *Continuum Mechanics and Thermodynamics* 10:189 ISSN 0935-1175 10.1007/s001610050089. [7] L. E. Silbert, et al. (2001) *Phys Rev E* 64(5):051302 doi. [8] J. Ai, et al. (2011) *Powder Technology* 206(3):269 ISSN 0032-5910 doi.