STUDY ON RESONANT ORBITS AROUND ELONGATED CELESTIAL BODIES. XY. Zeng[1], BD. Fang[2], JF. Li[1], K. Zhai[1], HX. Baoyin[1], and Y. Yu[1], [1]School of Aerospace Engineering, Tsinghua University, 100084, Beijing, China, zxy0985@gmail.com, [2]Shanghai Satellite Engineering Research Institute, 200240, Shanghai, China, fangbd@126.com.

Introduction: The majority of asteroids in the solar system are old minor celestial bodies. Their states of lowest energy correspond to the pure rotation with respect to their principal axis of highest moment of inertia with a constant angular moment. Some comets with small amplitudes of the nutation can be also treated as pure rotating bodies by considering a particle in their vicinity for a short time. Given the gravitational field and the rotating state of the target body, one can obtain the dynamic equations of a test particle. The equations are usually expressed in the body-fixed frame since they are time invariant for a uniformly rotating body as

$$\ddot{\boldsymbol{r}} + 2\boldsymbol{\omega}\times\dot{\boldsymbol{r}} + \boldsymbol{\omega}\times\left(\boldsymbol{\omega}\times\boldsymbol{r}\right) = -\nabla U \tag{1}$$

where $\boldsymbol{r}$ is the body-fixed position vector from the center of the target body to the vicinal particle or spacecraft, $\boldsymbol{\omega}$ is the rotating angular velocity vector of the central body with respect to the inertial reference frame, and U is the gravitational potential which is time invariant in the body-fixed frame.

In this paper the simplified model of a rotating mass dipole is adopted to approximate the group of natural elongated bodies. The model was proposed by Prieto-Llanos and Gomez-Tiemo which is suitable to approximate the elongated bodies with homogenous mass distribution. Figure 1 illustrates the rotating mass dipole with its synodical reference frame *oxyz*. The profile of the asteroid 216 Kleopatra is shown to indicate the relationship between the approximate model and the natural elongated body as an intuitional example. The rotating mass dipole is consisted with two primaries m_1 and m_2, separated by a massless rod in a characteristic distance d. The total system mass is M where $M = m_1 + m_2$. Its angular velocity is the same as the target body aligned with axis oz with $\boldsymbol{\omega} = \omega\boldsymbol{z}$. The origin of the frame *oxyz* is at the barycenter of the body where axis ox is collinear with the two primaries pointing from m_1 to m_2. The axis oy completes the right-handed frame. Here, the plane *oxy* is the same as the equatorial plane.

To reduce the calculation effort, a canonical dimensionless unit system can be applied to Eq. (1). It assumes $\omega = 1$, the distance unit is d which is the separation of the two primaries, and the mass unit is M. Consequently, the time unit is ω^{-1} and the spinning period of the central body is 2π.

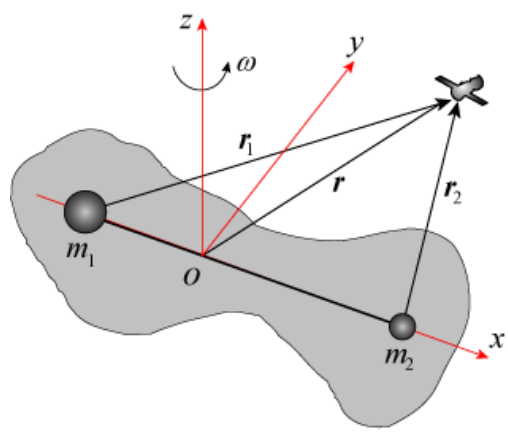

Figure 1. A schematic map of the rotating mass dipole and its synodic frame *oxyz*

After the above transformation, there will be a key parameter κ hereinafter referred to as the force ratio. It is actually the ratio between the gravitational force and the centrifugal force

$$\kappa = \frac{G\left(m_1+m_2\right)/d^2}{\omega^2 d} = \frac{GM}{\omega^2 d^3} \tag{2}$$

With appropriate parameters [μ, κ], potential distributions of different elongated bodies can be approximated. The relative tolerance of the locations of equilibrium points could be less than 5% for some asteroids, like 216 Kleopatra, 951 Gaspra, 1620 Geographos and even 103P/Hartley-2 (shown in Fig. 2).

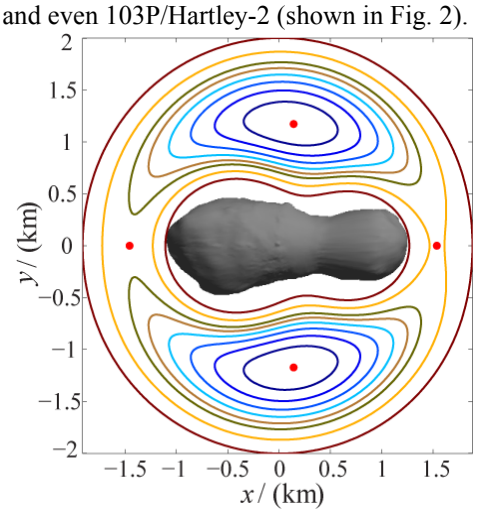

Figure 2. Zero-velocity curves and equilibrium points of the approximate models in the equatorial plane for 103P/Hartley-2

To obtain some common characteristics about the resonant orbit near elongated bodies, the unified approximate model is adopted. The orbital energy of a particle attracted by the rotating mass dipole can be defined as

$$\bar{E} = -\frac{1}{2}\left\|\dot{\bar{\boldsymbol{r}}} + \bar{\boldsymbol{\omega}} \times \bar{\boldsymbol{r}}\right\|^2 + \bar{U} = -\frac{1}{2}\left\|\dot{\bar{\boldsymbol{r}}} + \bar{\boldsymbol{\omega}} \times \bar{\boldsymbol{r}}\right\|^2 - \kappa\left(\frac{1-\mu}{\bar{r}_1} + \frac{\mu}{\bar{r}_2}\right) \tag{3}$$

The rate of energy change, i.e, the energy power, can be achieved with time derivative of Eq. (3) as

$$\bar{p}\left(\bar{\boldsymbol{r}}\right) = \frac{d}{d\bar{t}}\bar{E} = -\left(\bar{\boldsymbol{\omega}} \times \bar{\boldsymbol{r}}\right) \cdot \nabla \bar{U} \tag{4}$$

The above equation is a function relevant to the central gravitational field and the position of the particle. Figure 3 shows the field of this energy power in the equatorial plane with μ = 0.23 and κ = 6.64 which is similar to that of asteroid 951 Gaspra. Thus, a schematic projection profile of Gaspra is given in the figure. It is not say that this contour map is only suitable for 951 Gaspra. It actually indicates the map can be applied to realistic asteroids. Some common characteristics can be summarized as follows:

1) There are four equilibrium points around the central elongated body and all of them locate on the zero lines of $\bar{p}$. There are two perpendicular lines of $\bar{p} = 0$ intersected with each other at the point ($0.5 - \mu$, 0);

2) Four quadrants exist in the equatorial plane which are separated by the lines of $\bar{p} = 0$. They have been marked with I, II, III and IV in the figure. Based on the definition of Eq. (4), the value of $\bar{p}$ is negative in quadrants I and III whereas positive in II and IV. The absolute value $|\bar{p}|$ decreases rapidly along with the increase of the distance away from the body's surface;

3) No periodic orbits can exist in only one quadrant because the orbital energy monotonically increases in quadrants II and IV while monotonically decreases in quadrants I and III. To keep the periodic motion around the elongated body, the orbit should cover at least two quadrants with different signs of the energy power;

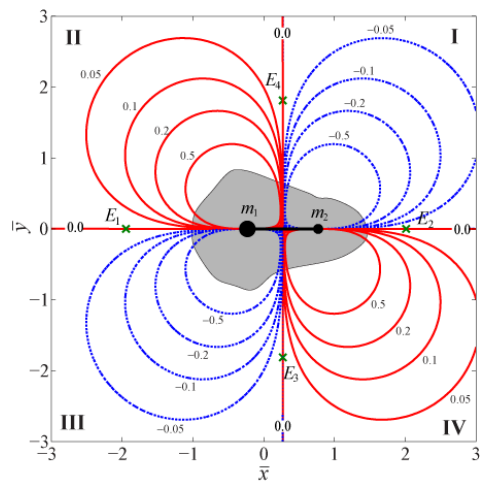

Figure 3. Contour map of the energy power for the rotating mass dipole with μ = 0.23 and κ = 6.64 (951 Gaspra)

4) At the far region away from the body's surface, the orbital energy is nearly constant and the energy power approaches zero. For example, at the distance $3d$ the value of $|\bar{p}| \approx 0.05$ is only about 3% of that near the body's surface with $|\bar{p}| \approx 1.5$ (which is not shown in the figure). It indicates that the impact of the irregular gravitational field on the orbit reduces at remote distances. In such cases, the orbit is similar to that under the two-body dynamic model. However, due to the weak attracting of the central potential, other perturbing forces like the solar gravity and the solar radiation pressure force should be considered.

More detailed analysis about the resonant orbit around elongated bodies will be given in the final presentation.

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