
Summary: Repeated cyclic loading can weaken many materials, causing them to experience brittle failure at much lower stresses than under static loading [1-5]. This process, called fatigue, is fundamental to engineering and may be highly relevant to icy satellites that experience cyclic tidal stresses during their eccentric orbits [6].

We hypothesize that under certain conditions, the near surface of an icy satellite can become fatigued due to diurnal tidal stresses, which could have a profound influence on ice shell rheology and geological evolution. Fatigue could explain why the surfaces of some satellites appear to fracture in response to tidal stresses [7-10] that are orders of magnitude lower than typical yield strength values of ice [11-13]. Here we describe the basic mechanisms of fatigue, develop an analytical model for the fatigue of an icy satellite surface, and discuss some of the possible geological consequences.

Introduction: The tensile strength of water ice is crucial in controlling what types of processes can fracture the surfaces of icy bodies. Laboratory experiments suggest a tensile strength at least 1 MPa [11-13]. Yet there is compelling evidence that the surfaces of some icy satellites are breaking in response to daily tidal stresses less than 100 kPa. For example, the timing of water vapor eruptions on Enceladus coincides with the timing of maximum tensile stresses from tides [8] and cycloids on Europa appear to form in response to daily tidal stresses [7,9-10]. One possibility that could resolve this discrepancy is that the yield strength of the surface may have been reduced as a result of fatigue from cyclic tidal forces.

The basic mechanism of fatigue is that microscopic flaws slowly grow into larger cracks under the action of cyclic stresses. These cracks concentrate stresses near their tips as $\sigma_{tip} \propto \sqrt{a}$, where $a$ is crack length [1]. The stress environment near the crack is described by the stress intensity $K_f = \sigma \sqrt{\pi a}$, where $\sigma$ is the far-field tensile stress perpendicular to the crack. Unstable crack growth (fracture) occurs when the stress intensity equals the fracture toughness $K_{fc}$ [14]. Fracture will then occur when a crack reaches the critical length $a_{cr} = K_{fc}^2 / (\pi \sigma^2)$.

An important property of ice is that it has a very low fracture toughness compared to other geologic materials (this is why you can break ice with your teeth), with $K_{fc} = 80 - 120$ kPa m$^{1/2}$ [15-16]. Once failure is reached, a specimen is said to be fatigued.

Approach: To ascertain whether an icy satellite surface can become fatigued, it is important to know: 1) the initial length of microcracks, 2) the magnitude of the daily tidal stress, and 3) the rate at which microcracks will grow. Here we assume that the initial crack length $a_0 \approx d$, the grain size, which may be 1 – 10 mm for convecting ice shells [17]. The magnitude of daily tidal stresses is a strong function of planet-satellite distance as well as the Love number $h_2$, which describes the radial deformation of the satellite in response to a tidal stress at harmonic degree 2 [18]. For Europa, the magnitude of the daily tidal stress ~50 kPa [7,19].

Laboratory experiments on fatigue suggest that the growth rates of microcracks in most brittle material conform to the Paris Law [1-3],

$$\frac{da}{dN} = C(\Delta K_i)^m$$

where $N$ is the number of loading cycles, $\Delta K$ is the change in stress intensity over one cycle and $C$ and $m$ are empirical constants. No direct measurements of $C$ and $m$ exist for water ice, but microcrack growth rates of other brittle geologic materials can be used to constrain these values. For example, wet quartz was found to have microcrack growth rates of $m = 15$ and $C = 10^{17}$ MPa$^{-m} m^{1+m/2}$ [20].

With estimates for these parameters, one could determine how long it would take a small flaw in an ice shell to grow from its initial length to the critical length for fracture. In other words, it is possible to calculate the fatigue lifetime of a satellite.

Preliminary Calculations: Here we consider $C$ and $m$ free parameters and determine the range of values that would lead to fatigue on the surface of Europa. In this simplified preliminary calculation, we assume an initial crack size of 10 mm, a tidal stress magnitude of 40 kPa, and presume that cracks will grow to the critical length for failure over a period of $10^6$ to $10^9$ years. Figure 1 shows the range of $C$ and $m$ that lead to the fatigue of the surface of Europa under these conditions. These results suggest that the surface of Europa could become fatigued if microcrack growth rates in ice are comparable to wet quartz. However, we emphasize that different microcrack growth mechanisms can operate in different materials under various conditions, so fatigue data on ice remain essential [1].

PRELIMINARY CALCULATIONS
mechanisms for the wide range in morphological features seen on the surfaces of icy moons.

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