EVALUATION OF DEGREE-1200 GRAIL GRAVITY MODELS USING LINE-OF-SIGHT DATA AND SPECTRAL ANALYSIS. Anton I. Ermakov¹, Maria T. Zuber¹, David E. Smith¹,², Ryan S. Park¹, Alex S. Konopliv³, Gregory A. Neumann², ¹Department of Earth, Atmospheric and Planetary Sciences, Massachusetts Institute of Technology, Cambridge, MA 02139, USA (eai@mit.edu), ²NASA/Goddard Space Flight Center, Greenbelt, MD, 20771, USA, ³NASA/Jet Propulsion Laboratory, Pasadena, 91101, USA.

Introduction: GRAIL (Gravity Recovery and Interior Laboratory) is NASA’s mission to determine the gravity field of the Moon with unprecedented accuracy [1, 2]. The GRAIL twin spacecraft were launched to the Moon on a low-energy transfer trajectory on September 10th, 2011. The primary mission was from March 1st to May 29th, 2012 with mean altitude of 55 km, the extended mission was from August 30th to December 14th, 2012 with mean altitude of 23 km. The gravity field model is recovered from the range data between the two GRAIL spacecraft in combination with the Deep Space Network (DSN) tracking for orbit determination. At the time of this writing, several degree and order 1200 (spatial blocksize = 4.5 km) gravity models were produced at the Jet Propulsion Laboratory. For this study, we focus on two gravity field models (later model A and B). The two models were produced with different constraints applied to the data. A looser constant spectral power constraint was applied to model A after degree 701. A tighter Kaula law was applied to model B after degree 701.

It is expected that at progressively higher spherical harmonic degrees (i.e. smaller spatial scale) the gravity signal is dominated by topographic variations as opposed to local density variations. The topography of the Moon is known to higher resolution than gravity due to mapping by the Lunar Orbiter Laser Altimeter (LOLA) [3]. The gravity-from-topography is computed using [4] to degree 1500 including 15 powers of topography. Consequently, the gravity associated with topography provides a useful reference for gravity model determination.

The goal of this paper is to evaluate the two degree-1200 GRAIL gravity models using spectral analysis and observed GRAIL LOS acceleration data. We compare models A and B against the gravity-from-topography by computing spectral estimates both globally and locally. This allows us to understand strengths and weaknesses of the models as well as to identify possible artifacts in the models, which will be useful in geophysical interpretations of the gravity models.

Global spectral analysis: First, we compute and compare the global spectral properties of the gravity and gravity-from-topography models. These spectral properties include spectral power, correlation, isotropic ratio and effective density spectrum. The isotropic ratio [5] is puzzling; it decreases at smaller spatial scales for both gravity models and gravity-from-topography-model. However, the isotropic ratio of lunar topography obtained LOLA [3] stays nearly unity.

Spectral-spatial localization: Second, we compute localized correlations between gravity and topography using spectral-spatial localization method with Slepian functions [6]. This parameter, in combination, with the LOS magnitude squared coherence, allows characterization of how the gravity field model is correlated with topography in spatial and spectral domains. In order to find localization centers, we used icosahedron tesselation of a sphere. The use of icosahedron tessellation has two main advantages: it is easily implementable in a recursive procedure, and it produces quasi-uniform sampling of a sphere as opposed to latitude/longitude grid sampling. The gravity and gravity-from-topography fields were localized in spherical cap windows with a radius of 9 degrees. We find that the correlation coefficient starts to decrease at degrees between 500 and 800, first in the maria regions and South Pole- Aitken (SP-A) basin, and later in the highlands.

Arc-per-arc analysis of magnitude-squared coherence: Third, we compute the magnitude-squared coherence for the LOS acceleration time series produced by the gravity models and by the gravity-from-topography model on arc-per-arc basis. We computed the range-acceleration for the primary and extended missions using the gravity and gravity-from-topography models. We compute the magnitude-squared coherence for the two time series using the Welch’s modified periodogram method [7]. The coherence was computed for a segment length of 4096 samples using the Hamming window and 50% overlap. We find strong contributions of resonances in the coherence spectrum. The resonances cause drops in the coherence spectrum that occur at spherical harmonic degrees that correspond to integer multiples of spacecraft separation. The resonant degree \( n_{res} \) can be found with the following formula: \( n_{res}=2\pi Rm/d \), where \( R \) is the radius of the Moon, \( d \) is the spacecraft separation and \( m \) is a positive integer (resonance order). The differential range rate measurement is not sensitive to the resonant harmonics since the two spacecraft experience identical pulls from the resonant terms of the gravity field spherical harmonic expansion. Figure 1 shows an example of the coherence spectrum computed for the 08-NOV-2012 orbital arc for two gravity models produced with different constraints on the power spectrum.
Conclusions: Our analysis revealed several aspects of gravity determination that are important for geophysical interpretation of the gravity models. The looser constrained model A power spectrum follows the gravity-from-topography spectrum more closely. However, the correlation of the model A with the gravity-from-topography is worse than for tightly constrained model B. The effective density spectrum decreases substantially for the model B, whereas it follows a linear trend for model A. Interestingly, the isotropic ratio decreases for the gravity-from-topography model (i.e. more East-West variations), as well as for both GRAIL gravity models. This happens in response to the nonlinear nature of the gravity due to topographic effects (see Eq. 10, 15-16 in [4]). On the other hand, the isotropic ratio of lunar topography stays nearly unity. Analysis of how isotropic topography can produce anisotropic gravity is currently ongoing. The correlation coefficient starts to decrease at degrees 500-800, first in the maria and SP-A regions and later in the highlands. This, along with degree-strength maps, is useful for determining the optimal location-dependent degree of truncation of the gravity models. For certain arcs the coherence spectrum drops substantially at degrees corresponding to integer multiples of the spacecraft separation. This should be taken into account when using LOS acceleration data for studying and interpreting small-scale (half-wavelengths of 4.5-10 km) gravity anomalies.


Figure 1. Magnitude-squared coherence for the 08-NOV-2012 arc. Top: The green vertical lines correspond to resonant degrees of orders from one to eight for the mean spacecraft separation (55.8 km). The yellow vertical lines indicate the location of the resonance for the minimum and maximum spacecraft separation. The blue curve and red curve are coherences for gravity model A and B, respectively. Bottom: Map of the lunar topography LOLA [3]. The blue curves show the spacecraft trajectory projected onto the lunar surface.