RELEVANCE OF THE 2ND LAW, GRAVITATIONAL POTENTIAL, ROTATION, AND HEAT TRANSFER TO CORE FORMATION AND DIFFERENTIATION OF PLANETS AND MOONS.
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Introduction: Rocky planet thermal history depends on several factors: initial conditions; the amounts, types, and distribution of heat sources; the materials present and their heat transport properties; and magmatism and degassing, which outpace convection and conduction because latent heat is large and the phases involved are rapidly buoyed upwards. Cooling is strongly affected by locations of heat-emitting elements and magma generation, which in turn create compositional layers, and so planetary thermal, physical, and chemical evolution are inseparable [1]. Existing chemical models are clearly deficient in carbon, suggesting key processes were overlooked. We evaluate the roles of gravity, entropy-energy, rotation, and co-accretion of ices in differentiation of the Earth, and use the present radioactive emissions to derive a bulk composition that is consistent with oxygen isotopes and compositions of meteorites and their inclusions [1,2,3]. Our approach can be extended to other bodies.

Extreme Temperatures in a Homogenous Planet: Temperatures in a sphere in steady state with internal heat production are governed by Fourier's equation in radial coordinates. Taking Earth as an example with its current measured global power of 30 TW (=AV_{Earth}) and current average thermal conductivity k=6 W/(m-K), a two shell model estimates central temperature as:

$$T_{\mathrm{center}} \sim [Aa^2/6k](a-b)/a \sim 30{,}000(a-b)/a \qquad (1)$$

where a is the surface radius and b is the radius below which radioactivity is lacking [1]. As the planets accreted, their internal temperatures rose, causing melting and degassing. Density differences stratified iron, oxides and silicates in the gravitational field (Table 1). The degree of differentiation depends on the body size, since this determines T_{center} and gravity, see below.

Effects of Ice incorporation on Differentiation: Solar abundances suggest that Earth's carbon content has been grossly underestimated. Not only is CO abundant in molecular clouds (along with CO_2, CH_4, and H_2O), but comet mineralogy implicates that other primitive accreted objects were mixture of dusts and ice.

During assembly of the planets, heating promoted reactions between Fe metal, silicate and oxide dusts, and trapped CO ice. Reactions with end-member silicates such as forsterite and enstatite are possible since these phases are identified in circumstellar environments (Molster et al., 2001). Examples after [2,3] are:

$$4Fe + CO \rightarrow Fe_3C + FeO \qquad (2)$$

$$FeO+MgSiO_3 \text{ (enstatite)} \rightarrow (Mg,Fe)SiO_4 \text{ (olivine)} \qquad (3)$$

About 10% C in the core balances 10%Fe in upper mantle olivine. In the absence of Fe metal, graphite would be produced via related reactions.

Oxides and silicates would be separated from metals in accord with known immiscibility of metal and silicate liquids. H_2O would be driven upwards, rather than producing metal hydrides, due to low densities and low melting points.

Heating would have violently and rapidly released unknown amounts of unreacted ices, reducing its C content, with the exception of C dissolved in the core or retained in the mantle as the dense phase diamond.

Energy-Entropy Effects of Core Formation: Core formation produces internal order, requiring loss of heat to space [2]. The relevant equation is:

$$\Delta U_{\mathrm{g}} = -\Delta \mathrm{R.E.} + S_{\mathrm{f}}\Delta T + T_{\mathrm{i}}\Delta S \qquad (4)$$

where S is entropy (i.e., uncompensated heat). For several reasons, internal T rises insignificantly. Foremost, because ΔU_{g} is negative per Newton's law, and if no other changes occur, ΔT must be negative. Positive ΔT is only possible if the other terms on the right side of Eq. 4 are much more negative than ΔU_{g}.

Because high T exist early on (Eq. 1), the $T\Delta S$ term is large. Estimates of unmixing [2] provide energy similar to ΔU_{g}. Similarly, large ΔR.E. can be produced via differential rotation of the core and mantle [2]. Friction in a liquid is low and cannot provide for copious heating which has the wrong sign to offset changes in U_{g}.

Within a convecting early Earth, bouyant hot silicate particles rose at a faster rate than heavy Fe particles in the polar upwellings. Ascending silicate grains would carry no angular momentum upwards from the core region which creates differential rotation [2]. This would be attenuated with time.

Gravity (from planet size) limits differentiation: Small planets are cold relative to large (Eq. 1, left) which limits their potential for differentiation via convection. Their gravitational segregation is even more limited. Mass fractions of planetary cores depend on gravitational acceleration (g) and are associated with a minimum surface g ~1.2 ms^{-2} (Fig. 1). An object must have a mass >~10^{22} kg to have a metallic core, which is greater than the mass of the entire asteroid belt.

This behavior is consistent with Stoke's settling velocity of grains of diameter d in boundary layers, at

$v = g\Delta\rho d^2/(18\eta)$, since the viscosity decreases with T, and both g and T depend on planet size [3].

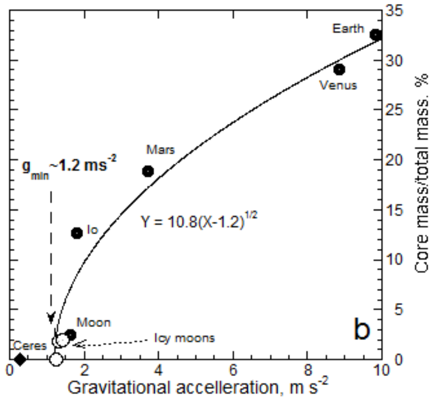

Fig. 1. Dependence of the ratio of core mass to total planetary mass on the surface gravitational acceleration, g [3]. Dashed vertical arrow emphasizes the minimum g-value needed for core formation.

A New Meteorite Mixing Model for Earth's Chemical Composition: We propose a mixing model [1]. Our approach avoids problems in previous estimates, such as assuming large, preferential loss of volatile elements, while treating refractory and lithophile elements in different ways. We account for oxygen isotope data on solar system materials, which require that Earth includes two non-metallic meteoritic reservoirs, one of which is high in refractory elements [3].

To ascertain Earth's radionuclide concentrations, we calculate the power emitted as a function of U and K contents (Fig. 2). Emissions of Th are combined with U since the Th/U ratio is about 3.75 in Solar System materials. Earth lies on the global power = 30 TW line at its geochemically determined K/U ratio of 10^4.

One presolar reservoir is iron metal, devoid of radioactive elements (required by Eq. 1). Because the chondritric reservoir had far too much K compared to U to represent Earth, a meteorite reservoir with substantial U and very little K must have existed. Earth's bulk composition must lie inside the triangle defined by these three reservoirs. Because the bulk Earth is ⅓ iron (the core has ~10 wt % light element, whereas Fe constitutes ~10 wt% of the silicate portion) and the remaining 2/3 of the material which now contains all the U, Th, and K, the modern bulk oxide-Earth (BOE) must lie on the "45 TW" line of Fig. 2. Therefore, both non-metallic meteoritic reservoirs must lie along a tie line that pivots about the BOE point. Ordinary chondrites define a mixing line of silicates with iron.

The third reservoir must lie below the K/U~10^4 line. We place this on the K = 0 line and denote it "refractory" because U and Th, but not K, are concentrated in such materials. Remnants of the refractory reservoir exist as calcium-aluminum inclusions and other material in meteorites, which contrasts with the silicate reservoir, which contains both K and U. The existence of short-lived isotopes such as ^{26}Al is consistent with the refractory reservoir having been injected into the pre-solar nebula.

Divisions of the zones in the Earth (Table 1) have nearly the same percentages as those independently inferred for mineral reservoirs in Figure 2.

Knowledge of heat emissions of other rocky planets would similarly constrain their compositions.

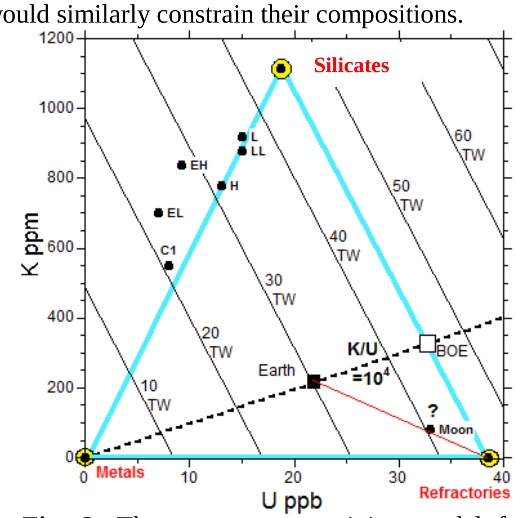

Fig. 2. Three component mixing model for the Earth, defined by geochemical, geophysical, and meteoritic constraints, after [1]. Contours of global power are shown for a body of Earth's mass having the indicated elemental compositions, presuming Th/U = 3.75. Labeled dots show the K vs. U for chondrite classes and a possible Moon composition [4].

Table 1. Dust Reservoirs cf. Zones in the Earth.

Zone	Mass %	Reservoir	Elements	Density g cm^{-3}
Core	33	Metal	Fe, Ni, C>S, N, P	~6.8
LM	50	Refractory	O,Ca,Al,Mg,Si>Fe,Ti	~3.8
UM*	17	Silicate	O,Mg,Si,Fe>Ca,Na,Al,K*	<3.0

*Includes lithophiles such as U and Th, many of which originated in the refractory reservoir but were carried upwards early on by magmas.

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