MATLAB PROGRAM TO CONSTRUCT BOUGUER GRAVITY ANOMALY FIELD USING ULTRA HIGH DEGREE SPHERICAL HARMONIC COEFFICIENTS. H. W. Yang[1], W. J. Zhao[2], Z. H. Wu[3], [1,2,3]Chinese Academy of Geological Sciences, 26 Baiwanzhuang Road, Beijing, 100037, China, Email: [1]yhw1106@163.com;

Summary: Summation of ultra high degree spherical harmonic expansion is one essential process to construct planetary gravity models based on satellite data. Specially, SHTOOLS software package is a Fortran-based package for transformation between the ultra high degree spherical harmonic expansion and grid-node style gravity values on planetary surface. For the convenience and further research, the summation in Matlab code is build in this paper. Compared to Fortran language, advantages in Matrix Processing supported by Matlab software will effectively reduce the complexity and length of the program. Moreover, Matlab program for Bouguer correction is also introduced here, derived from high precise topographic data and free air gravity anomaly from ultra high degree spherical harmonic expansion mentioned above. Also, distributed parallel computation on Linux platform supported by new version Matlab software can improve efficiency of computation. These programs provide an important and fundamental approach to study gravity models of any planets, including Earth, Moon, Mars, Venus and etc.

Introduction: Because of the limited budget, gravity models based on satellite tracking data or satellite gravity data become the most effective geophysical observations to study interior of planets. The sets of spherical harmonic coefficients of planetary gravity models are fundamental products derived from satellite observations. As the further needs increased on high accurate gravity model, when calculation of ultra high degree spherical harmonic expansion was implemented , the approaches and program packages to solve the difficulties in IEEE underflow and overflow in this process had became an inevitable problem. The common solution to this problem is Clenshaw's method, but Holmes and Featherstone[1] constituted new modified forward methods to evaluate these expansions, which has advantages in simplicity of formulation over and similar stability, precision and calculation efficiency as Clenshaw's. In this paper, these modified approaches have been compiled to construct free air gravity anomaly field from ultra high degree (up to 2700) spherical harmonic coefficients in Matlab, which has more powerful Matrix Manipulation than Fortran in SHTOOLS. The flow chart of the Matlab program and its advantages will introduced in this paper. Moreover, the computation of Bouguer gravity anomaly field reduction based on the free air gravity field derived above and topographic data will also introduced. Distributed parallel computation capability in Matlab on Linux platform is essential to improving the numerical efficiency.

Principals and Methods: The gravity anomaly △g can be expressed in a spherical harmonic expansion:

$$\Delta g(r,\lambda,\varphi)=\frac{GM}{r^2}\sum_{n=2}^{N}\left(\frac{R_e}{r}\right)^n(n-1)\sum_{m=0}^{n}\bar{P}_{nm}(\sin\theta)\left(\bar{C}_{nm}\cos m\lambda+\bar{S}_{nm}\sin m\lambda\right)$$

For convenient computation, the alternative expression can be written as

$$\Delta g(r,\lambda,\varphi)=\frac{GM}{r^2}\sum_{m=0}^{M}\left[\cos m\lambda\sum_{n=\mu}^{M}\left(\frac{R_e}{r}\right)^n(n-1)\bar{C}_{nm}\bar{P}_{nm}(\sin\theta)+\sin m\lambda\sum_{n=\mu}^{M}\left(\frac{R_e}{r}\right)^n(n-1)\bar{S}_{nm}\bar{P}_{nm}(\sin\theta)\right]$$

The most effective computation process should be partial summations at first of fully normalized functions as *n* increase (Step 1~5 in Fig.1), and then subsequent summations functions as *m* increase, and final results(step 6). There are 3 principal problems in this process, which also determine the maximum degree in this spherical harmonic expansion. (1) The function to calculate $\bar{P}_{mm}(\sin\theta)$ can be written as $\bar{P}_{mm}(\sin\theta)=u^m\sqrt{3}\prod_{i=2}^{m}\sqrt{\frac{2i+1}{2i}}(m\geq 1)$, where $u=\sin\theta$. For ultra high value of degree (above 300), specially at high latitude close to the poles ($\theta\to 0^0$ or $\theta\to 180^0$), u^m will range over thousands of orders of magnitude and cause the computation underflow in IEEE double precision. Computation the quantities $\bar{P}_{mm}(\sin\theta)/u^m$ was found by Libbrecht[2] as an effective method to avoid this problem. The subsequent computations of $\bar{P}_{nm}/u^m$ could also be yielded by recursive algorithm based on the relations between $\bar{P}_{mm}$ and $\bar{P}_{nm}$. (2) Meanwhile, as the entire ranges of $\bar{P}_{nm}/u^m$ lead to overflow problem in IEEE double precision, the global scale 10^{-280} will be used to multiplying the all values of $\bar{P}_{nm}/u^m$. Actually, the maximum degree of 2700 is determined by the sectoral values of $\bar{P}_{nm}/u^m$. (3) To avoid eliminating the tiny variance from normalized functions at high degree, the summation should start from the high degree to the low degree. All summation process will be implemented with matrix in Malab. Improvement of the performance benefits from the matrix processing and some in-built functions supported by Matlab.

Contract to Bouguer reduction in regional scale, curvature corrections of a planet and spherical coordinates will be taken into consideration for high accurate reduction in global. Therefore, tesseroid mass elements have been approved to be most suitable for Bouguer reductions with highest computation efficiency using topographic data and free air gravity anomaly field derived above[3,4] (Step 7 in Fig.1). Bouguer reduction, which is a time-consuming operation, has been significantly in further improved by distributed parallel Matlab programming codes on Linux platform (Step 8 in Fig.1).

Flow Chart of Matlab Program: Here is the flow chart of the Matlab program to construct Bouguer anomaly gravity field using data sets of spherical harmonic coefficients (SHC).

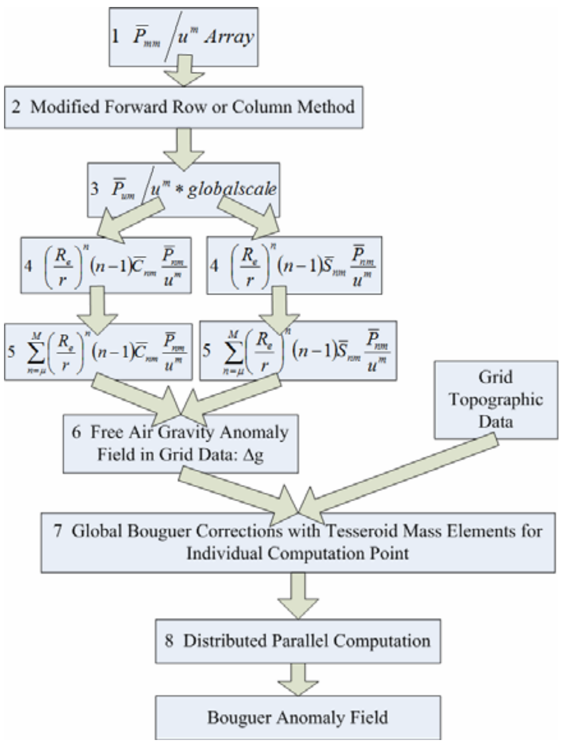

Fig.1 Flow chart of Bouguer anomaly gravity field computation from SHC (spherical harmonic coefficients) in Matlab

Results:
As the highest degree of recent spherical harmonic model of the Moon is 660 (equal to 8km scale, Fig.2), the digital elevation model with similar spatial interval (4 pixel per degree) has been chosen to construct Bouguer gravity anomaly map (Fig.3). Similarly, the most accurate free air gravity anomaly model can be built with EGM2008 at the degree 2190 (Fig.4).

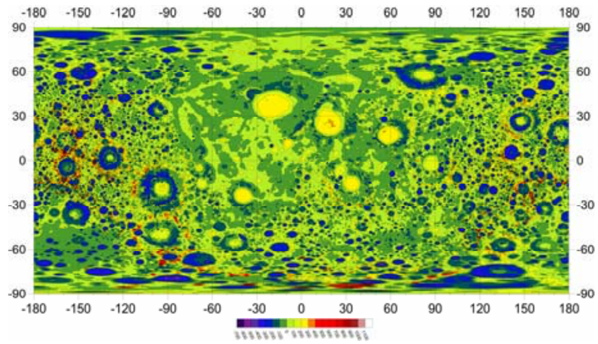

Fig. 2 Free Air Gravity Anomaly Field of the Moon (gggrx_0660pm_anom_l320)(=8km space interv)

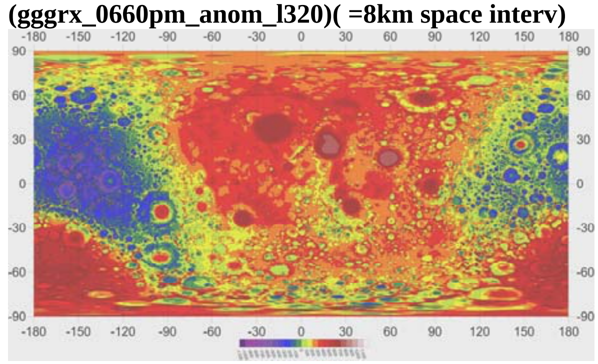

Fig. 3 Bouguer Gravity Anomaly Field of the Moon (gggrx_0660pm _anom_l320, and ldem4)

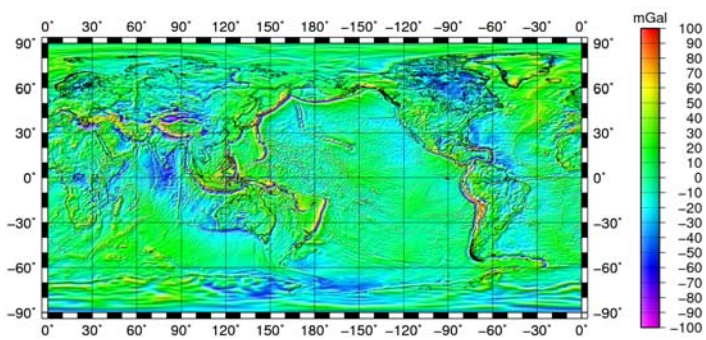

Fig.4 Free Air Gravity Anomaly Field of the Earth (EGM2008 at degree 2190)

References: [1] S. A. Holemes and W. E. Featherstone, J. Geodesy (2002) 76: 279; [2] K. G. Libbrecje, Solar Phys. (1985) 99: 371; [3] B. Heck, K. Seitz, J. Geodesy (2007) 81:121-136; [4] F. Wild-Pfeiffer J. Geodesy (2008) 82: 637-653;