THE CLOSED TRAJECTORIES OF UNDISCOVERED SMALL BODIES IN THE SOLAR SYSTEM. N. I. Perov, Cultural and Educational Centre named after Valentina Tereshkova, ul. Chaikovskogo, 3, Yaroslavl, 150000, Russian Federation. E-mail: perov@yarplaneta.ru.

Introduction: There are known 700000 celestial bodies and almost all of them have been discovered with using optical observations [1]. Existence only several bodies or group of the bodies has been predicted by theoretical way (Neptune, Trojans). Below, we consider the region of motion of a particle with negligible small mass \( m_3 \) in the frame of the planar circular restricted three body problem [2], [3]. Let us, \( m_1 \) and \( m_2 \) are mass of main bodies, \( r_{12} \) is a distance between these bodies, and \( G \) is the gravitational constant. We find the region of the point motion, – distance \( r_3 \), \( (r_3=x_3,y_3) \) in respect of the system center mass, – and numerically investigate the region of the particle stability motion, using method of Runge-Kutta integrating, where \( N \) is the numbers of points in the figures. The regions of the particles stable motion, in the given model, will be considered as the regions of concentration of undiscovered bodies in these systems.

Fundamental Equation: In accordance with works [2], [3] and [4] we have the vector differential equation (1) of the particle \( m_3 \) motion in the uniformly rotating system

\[
d^2r_3/dt^2+Gm_1(r_3-r_1)/( (r_3-r_1)^3) + Gm_2(r_3-r_2)/( (r_3-r_2)^3)-2[d\mathbf{r}/dt, \Omega]-\Omega^2r_3 = 0. \tag{1}
\]

Here, \( r_3 \) is the radius-vector determined the position of considered point in respect of the center mass of the system. \( r_1 \) and \( r_2 \) are radii – vectors in respect of the center mass of the system determined the positions of major bodies with mass \( m_1 \) and \( m_2 \) correspondingly. \( \Omega \) is the angular velocity of uniformly rotation of the major bodies.

\[
r_1=-(m_1/(m_1+m_2))r_{12}, r_2=(m_1/(m_1+m_2))r_{12}. \tag{2}
\]

\[
\Omega = \sqrt{G(m_1+m_2)/r_{12}^3}.
\]

Examples: For the numerical experiments we put \( G=6.672 \times 10^{-11} \text{ m}^3/(\text{kg sec}^2) \), \( m_1=2 \times 10^{10} \text{ kg} \) (mass of the Sun), \( m_2 \) is mass of a planet. In the process of the equation (1) solving we use the following units: \( m_1 \) is the unit of mass, \( r_{12} \) is the unit of length, the unit of time \( t \) is corresponded for the case \( G=1 \). Moreover, we put for all considered cases the following initial conditions: \( x_1=0, dy_1/dt=0, y_1=0, dx_2/dt=0, x_2=0, dy_2/dt=0, y_2=0, dy_3/dt=0, x_3=0, dy_3/dt=0, y_3=0, dy_3/dt=0 \). The results of the numerical experiments in intervals of time motion corresponded hundreds and thousands revolutions of major bodies are presented in Fig. 1 – 5.

Conclusions: a) For the curves (Fig.1. – Fig.5.) the velocity of \( m_3 \) equals zero only in initial moment of time but in the work [2] the corresponding curves are plotted, mainly, only for \( V=0 \). b) Asteroid 2010 SO16 moves along the trajectory like presented in Fig. 1. (Fig. 4.) and it approaches the Earth [5], so, it may be supposed – there are undiscovered small bodies in the systems “the Sun and a major planet” moving along the curves presented in Fig. 1. and Fig.4. c) In Fig. 2. illustrated “strange” closed trajectories of small bodies in the system “the Sun and a major planet”, the regions for for casting of undiscovered small bodies, moved in main belt of asteroids, Kuiper belt and in the systems “the Sun and Saturn” and “the Sun and Uranus” are pointed out. d) Fig. 3 and Fig. 5. are not contradicted with the celestial mechanical model of some main belt asteroids, some Kuiper belt objects and some short periodical comets origin (transfers from planet centrically to heliocentrically orbits and vice – verse).

The closed trajectory of the small body $m_3$ in the system “The Sun ($m_1=1$) and Jupiter ($m_2=m_1/1048$)”, in respect of the center mass of the system ($m_1$ and $m_2$). $x_1=1/1049$, $x_2=1048/1049$. $x_{30}=-x_2+\epsilon$, $\epsilon=0.0983$. Unit of length equals $r_{12}$. Time of $m_3$ motion equals $\tau=2000$ units of time. $\Omega=274838^{1/2}/524$. ($\Omega=1$).

The “closed” trajectory of the small body $m_3$ in the system “The Sun ($m_1=1$) and Pluto ($m_2=m_1/1.35\times10^3$)”, in respect of the center mass of the system ($m_1$ and $m_2$). $x_1=-7.40740740\times10^{-9}$, $x_2=0.999999999259259$. $x_{30}=-x_2+\epsilon$, $\epsilon=1\times10^{-4}$. Unit of length equals $r_{12}$. Time of $m_3$ motion $\tau=16000$ units of time. $\Omega=1.0000000003703703$. ($\Omega=1$). Number of points is equal to 160000.

The “closed” trajectory of the small body $m_3$ in the system “The Sun ($m_1=1$) and Neptune ($m_2=m_1/19412$)”, in respect of the center mass of the system ($m_1$ and $m_2$). $x_1=-1/19412$, $x_2=19412/19413$. $x_{30}=-x_2+\epsilon$, $\epsilon=1.024$. Unit of length equals $r_{12}$. $\Omega=(3/9706)\times10^{467921^{1/2}}$. ($\Omega=1$).