Scaling Laws of Impact Induced Shock Pressure and Particle Velocity in Planetary Mantle

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1. Introduction

Planets are formed by accreting a huge number of small planetesimals, a few km to a few tens of km in size, in the solar nebula. Hundreds of thousands of collisions must have occurred during the formation of small planets such as Mercury and Mars when they were orbiting the Sun inside a dense population of planetesimal.

The shock wave produced by a large impact increases the temperature in the mantle and the core of the planets directly beneath the impact site, enhancing mantle convection [*e.g., Watters et al., 2009; Roberts and Arkani-Hamed, 2012,2014*], modifying the CMB heat flux, which could in turn favour a hemispheric dynamo on Mars [*Monteux et al., 2015*], or crippling the core dynamo [*Arkani-Hamed and Olson, 2010*].

The impact-induced shock pressure inside a planet can be investigated by numerically solving the shock dynamic equations using hydrocode simulations, which demand considerable computer capacity for one single impact. Hence, the scaling laws derived from these models are of great interest when considering the full accretionary history of a planetary objects.

Here we model shock pressure and particle velocity distributions in the mantle using hydrocode simulations for impact velocities of 4 to 10 km/s and projectile diameters ranging from 100 to 400 km, as an attempt to extend *Pierazzo et al.'s [1997]* scaling laws to low impact velocities and reasonable impactor radii occurring during the formation of terrestrial planets.

2. Hydrocode models of shock distribution

To model the thermo-mechanical evolution during an impact between a differentiated Mars size body and a large impactor, we use the iSALE-2D hydrocode, a multi-rheology, multi-material hydrocode, specifically developed to model impact crater formation on a planetary scale *[Collins et al., 2004, Davison et al., 2010]* (**Tab. 1**).

Immediately after the impact, the shockwave propagates downward from the impact site. The front of this shockwave reaches the core-mantle boundary in less than 5 minutes while the transient crater is still opening at the surface (**Fig 1**).

Table 1: Typical parameter values for numerical hydrocode models

Model Planet radius	3400 km
Model Planet core radius	1700 km
Impactor radius	50 - 200 km
Impact velocity	4 – 10 km/s
Mantle properties (silicates)	
Initial density	3314 kg/m^3
Equation of State type	ANEOS
Poisson	0.25
Core properties (Iron)	
Initial density	7840 kg/m^3
Equation of State type	ANEOS
Poisson	0.3

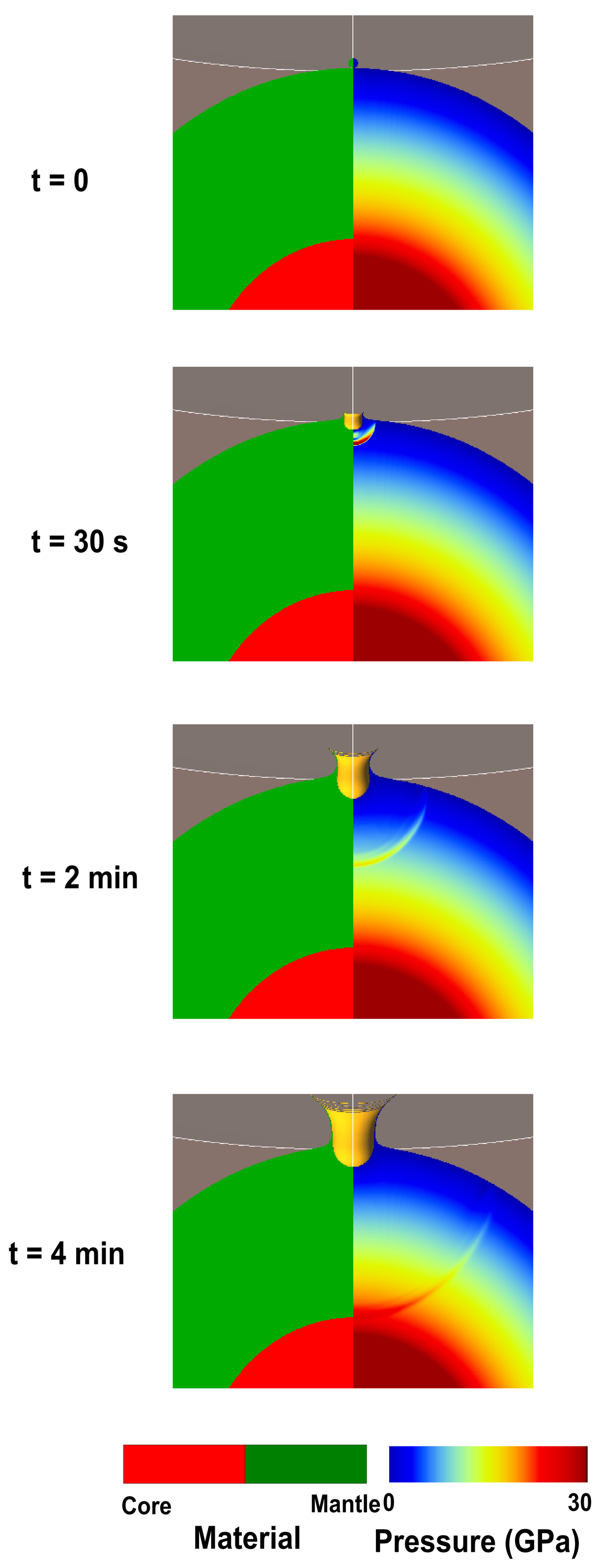

Figure 1: Close up view of the material repartition (left column) and total pressure (right column) as a function of time (from top to bottom) on the model planet (for V_{imp}=10 km/s and D_{imp}=100 km). The total pressure is the pre-impact lithostatic pressure plus the impact–induced shock pressure. In this model, the grid resolution is 2 km in all directions. The silicate mantle as well as the impactor is made of dunite.

3. Shock pressure and particle velocity scaling laws at low impact velocities

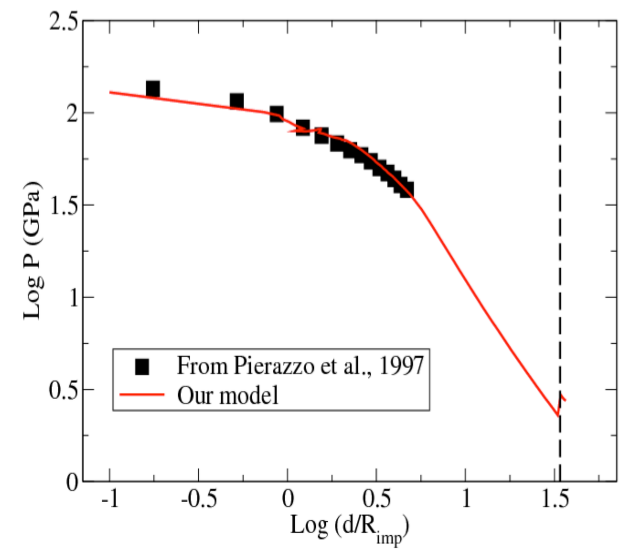

Figure 2: Peak pressure increase as a function of the depth normalized by the radius of the impactor. Here we only consider the case with R_{imp}=50 km and V_{imp}=10 km/s. The silicate mantle as well as the impactor is made of dunite. The result from our hydrocode model is represented by red line. Similar results from the model of *Pierazzo et al., (1997)* are represented with black squares. The vertical dashed line shows the core-mantle boundary of our model planet.

In **Fig. 2**, we monitor the peak pressure as a function of depth normalized by the impactor radius along the symmetry axis for the case illustrated in **Fig. 1**. This figure shows that our results are in agreement with the results obtained by *Pierazzo et al. [1997]* using a different hydrocode simulation.

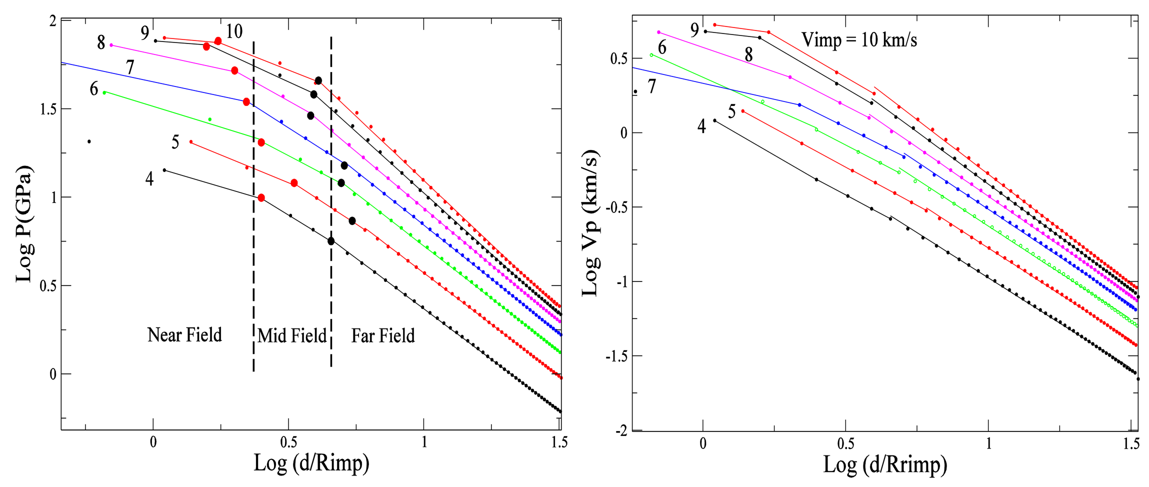

Figure 3 (left). Shock pressure versus normalized distance from the impact site at the surface for an impactor of 100 km diameter and impact velocities ranging from 4 to 10 km/s. The numbers on the curves are the impact velocities. The hydrocode results are presented by dots, while the regression lines to the near field, mid field and far field regions are straight lines. **3 (right).** shows the corresponding particle velocity.

From **Fig. 3**, we propose three distinct regions in the mantle:

- a near field regime (i=1), which extends to 1-3 times the projectile radius into the target, where the peak shock pressure and particle velocity decay very slowly with increasing distance,
- a mid field region (i=2), which extends to ~ 4.5 times the impactor radius, where the pressure and particle velocity decay exponentially but moderately,
- a more distant far field region (i=3) where the pressure and particle velocity decay strongly with distance.

From the hydrocode models and using the following scaling laws:

$$\text{Log } P_i = a_i + n_i \text{ Log}(d_i/R_{imp}), \quad i=1, 2, \text{ and } 3. \quad (1)$$

$$\text{Log } V_{pi} = c_i + m_i \text{ Log}(d_i/R_{imp}), \quad i=1, 2, \text{ and } 3. \quad (2)$$

we obtain the values of a_i, c_i, n_i and m_i in the 3 regions as a function of the impact velocity v_{imp} (**Fig. 4**).

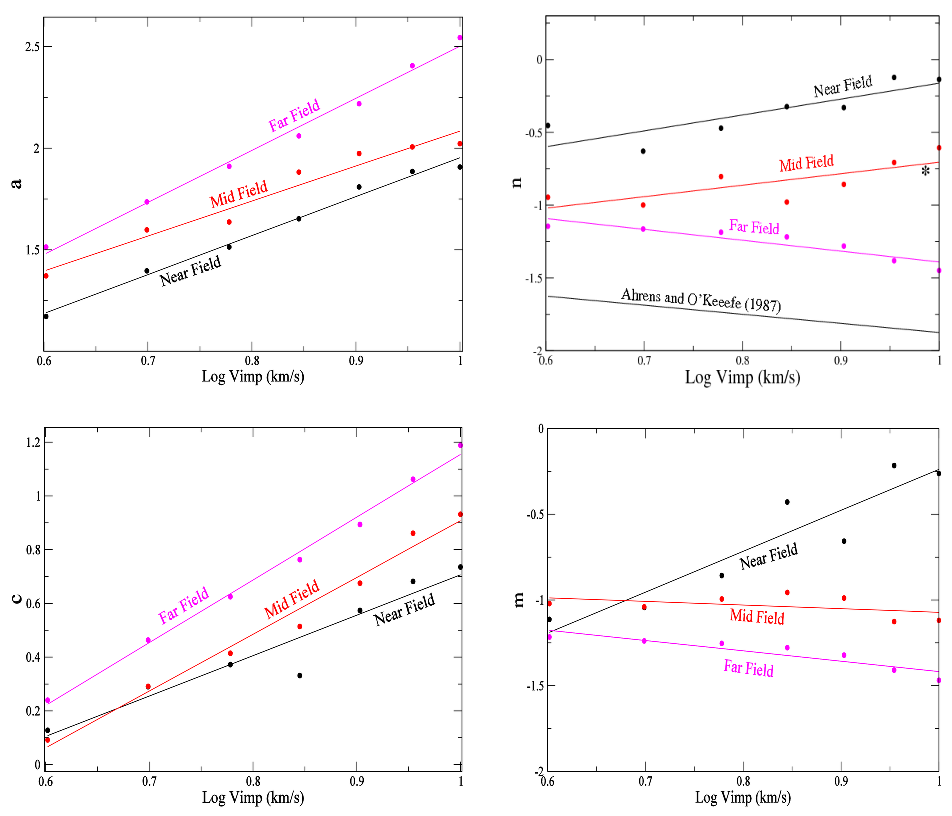

Figure 4 Dependence of regression parameters *a*, *n*, *c*, and *m* in Equations 1 and 2 on the impact velocity. Dots are based on hydrocode models and lines are regression fits

4. Modeling implications

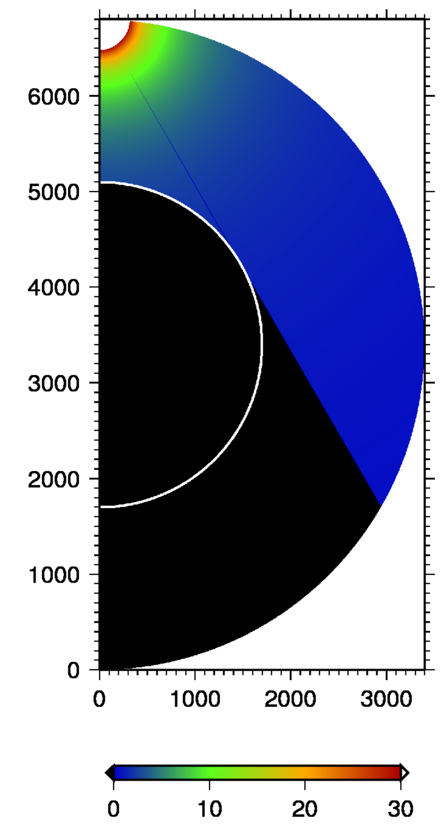

Figure 5: 2D distribution of the peak shock pressure determined for an impactor of 100 km in diameter and an impact velocity of 10 km/s calculated using the scaling laws in near field, mid field, and far field, and the parameter values from Table 2. The grid spacing is 2 km in radial direction and 0.03 degrees in the colatitude direction.

Fig. 5 shows the 2D distribution of the peak shock pressure determined for an impactor of 100 km in diameter and an impact velocity of 10 km/s calculated using our scaling laws in near field, mid field, and far field.

The entire computer time in a PC, CPU MHz: 2393.968, was only 16 seconds, which is substantially shorter than 48 hours taken by the corresponding hydrocode model using a CPU MHz: 2.9 GHz laptop (**Fig. 1**).

Fig. 6 shows the profiles of the pressure along the axis of symmetry for comparison. The differences between the hydrocode model and the ones calculated on the basis of the scaling laws are very minute.

Form these scaling laws we can now estimate the peak pressure increase and the particle velocities below the impact site for impact velocities 4 <v_{imp}< 10 km/s.

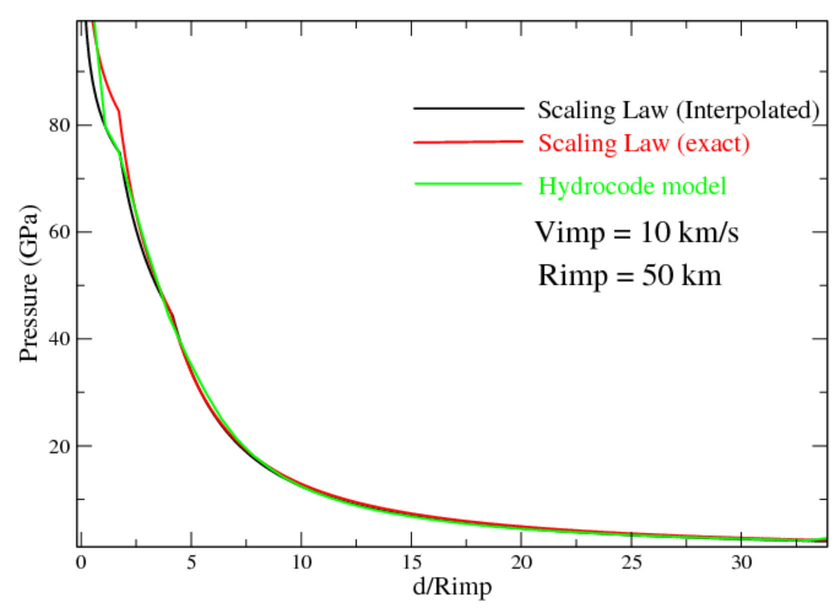

Figure 6. Shock pressure versus distance from the impact site normalized to the impactor radius. In the exact model we have used *a* and n from Equation (1), while regression parameters from Figure are used in the interpolated model

5. Conclusions

- We obtained scaling laws of the peak pressure increase and the particle velocities below the impact site for impact velocities ranging from 4 to 10 km/s, compatible with the accretionary conditions of terrestrial protoplanets.
- From these scaling laws we can now envision to estimate the temperature and melt volumes below the impact site

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