Study on Resonant Orbits around Elongated Celestial Bodies

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Abstract

Resonant orbits around natural elongated bodies are investigated to give some common characteristics in terms of the instantaneous orbital energy. The rotating mass dipole system is used to approximate the potential distribution of nearly axisymmetrical elongated bodies. The dynamical properties of the rotating mass dipole are briefly illustrated. The essential of the resonant effect and the 1:1 resonant orbit in the equatorial plane are illuminated analytically. Numerical simulations are presented to confirm the analytical discussions.

Common Features

To obtain some common characteristics about the resonant orbit near elongated bodies, the unified approximate model is adopted. The orbital energy of a particle attracted by the rotating mass dipole can be defined as

\[ E = \frac{1}{2} \nabla \cdot \nabla P = \frac{1}{2} \nabla \cdot \nabla (\mu P) \]

where \( \mu = \frac{m}{r^2} \) and \( P = \frac{GMm_1}{r^2} \) for a massless rod, connecting the two points as shown in Fig. 1. Two independent variables determine the potential distribution of the dipole system, that is, the mass ratio \( u \) and the force ratio \( k \). An example with equilibrium and zero-velocity curves is given in Fig. 2.

The above equation is a function relevant to the central gravitational field and the position of the particle. Figure 3 shows the field of this energy power in the equatorial plane with \( u = 0.23 \) and \( k = 6.64 \) (Approximation of the potential distribution of the asteroid 951 Gaspra).

The rate of energy change, i.e., the energy power, can be achieved with time derivative of the above equation as

\[ \frac{dE}{dt} = \frac{1}{2} \nabla \cdot \nabla (\mu P \nabla E) \]

The above equation clearly shows the essential of the 1:1 resonant orbit. The reason for the energy variation is the continuous work of the irregular gravitational field on the convected velocity. Figure 4 presents the energy variation corresponding to the 1:1 resonant orbit.

Concluding Remarks

The resonant orbits in the vicinity of natural elongated bodies are investigated by using a unified approximate model of the rotating mass dipole. Taking the instantaneous orbital energy as a discriminator, four quadrants in the body-fixed frame are obtained according to the energy variation of the resonant orbit with its peripapsis locating in quadrants II or IV whereas decreases for the orbit whose periapsis is in quadrant I or III. The reason for the energy variation of the resonant orbit is the continuous work of the irregular gravitational field on the convected velocity.

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