Study on Resonant Orbits around Elongated Celestial Bodies

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Abstract

Resonant orbits around natural elongated bodies are investigated to give some common characteristics in terms of the instantaneous orbital energy. The rotating mass dipole system is used to approximate the potential distribution of nearly axisymmetrical elongated bodies. The dynamical properties of the rotating mass dipole are briefly illustrated. The essential of the resonant effect and the 1:1 resonant orbit in the equatorial plane are illuminated analytically. Numerical simulations are presented to confirm the analytical discussions.

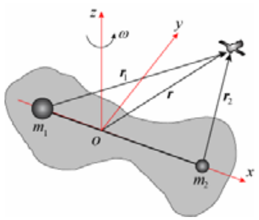


Fig.1 Schematic map of the rotating mass dipole and its synodic frame *oxyz*

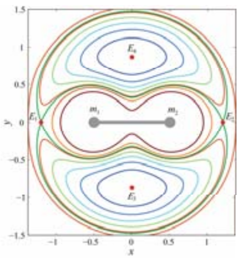


Fig.2 Equilibria and zero-velocity curves around the dipole with u = 0.5 and k = 1

Introduction

The rotating mass dipole is consisted with two point masses and a massless rod, connecting the two points as shown in Fig. 1. Two independent variables determine the potential distribution of the dipole system, i.e., the mass ratio u and the force ratio k. An example with equilibria and zero-velocity curves is given in Fig. 2.

Dynamical equations regarding the uniformly spinning minor bodies

$$\ddot{\boldsymbol{r}} + 2\begin{bmatrix}-\dot{y} & \dot{x} & 0\end{bmatrix}^{\mathrm{T}} = -\nabla V$$

where

$$V = -\frac{\left(x^2+y^2\right)}{2} - k\left(\frac{1-\mu}{r_1}+\frac{\mu}{r_2}\right) \quad \text{and} \quad k = \frac{G\left(m_1+m_2\right)/d^2}{\omega^2 d} = \frac{GM}{\omega^2 d^3}$$

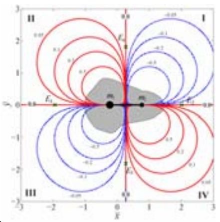


Fig.3 Contour map of the energy power for the rotating mass dipole with u = 0.23 and k = 6.64 (Approximation of the potential distribution of the asteroid 951 Gaspra)

Common Features

To obtain some common characteristics about the resonant orbit near elongated bodies, the unified approximate model is adopted. The orbital energy of a particle attracted by the rotating mass dipole can be defined as

$$\bar{E} = -\frac{1}{2}\left\|\dot{\bar{\boldsymbol{r}}} + \bar{\boldsymbol{\varpi}}\times\bar{\boldsymbol{r}}\right\|^2 + \bar{U} = -\frac{1}{2}\left\|\dot{\bar{\boldsymbol{r}}} + \bar{\boldsymbol{\varpi}}\times\bar{\boldsymbol{r}}\right\|^2 - \kappa\left(\frac{1-\mu}{\bar{r}_1}+\frac{\mu}{\bar{r}_2}\right)$$

The rate of energy change, i.e., the energy power, can be achieved with time derivative of the above equation as

$$\bar{p}\left(\bar{\boldsymbol{r}}\right) = \frac{d}{d\bar{t}}\bar{E} = -\left(\bar{\boldsymbol{\varpi}}\times\bar{\boldsymbol{r}}\right)\cdot\nabla\bar{U}$$

The above equation is a function relevant to the central gravitational field and the position of the particle. Figure 3 shows the field of this energy power in the equatorial plane with u = 0.23 and k = 6.64 as an example. Some common characteristics can be summarized as follows:

1) There are four equilibrium points around the central elongated body locating on the zero lines of p. There are two perpendicular lines of p = 0 intersected with each other at the point of (0.5 – u, 0);

2) Four quadrants exist in the equatorial plane which are separated by the lines of p = 0. the value of p is negative in quadrants I and III whereas positive in II and IV;

3) No periodic orbits can exist in only one quadrant because the orbital energy monotonically increases in quadrants II and IV while monotonically decreases in quadrants I and III;

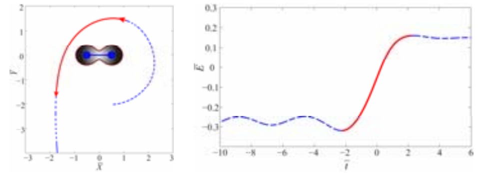
4) At the far region away from the body's surface, the orbital energy is nearly constant and the energy power approaches zero.

The 1:1 Resonant Orbits

The 1:1 resonant orbit is a temporary synchronization between the close orbit and the self-rotation of the central body. If the resonant period is t_s, (remembering that it is a short period), due to the very small magnitude of the relative velocity, the variation of the orbital energy can be approximated with

$$\Delta\bar{E} \approx \bar{p}\left(\bar{\boldsymbol{r}}_s\right)\cdot\bar{t}_s = -\left(\bar{\boldsymbol{\varpi}}\times\bar{\boldsymbol{r}}_s\right)\cdot\nabla\bar{U}\cdot\bar{t}_s = -\bar{\boldsymbol{v}}_e\cdot\nabla\bar{U}\cdot\bar{t}$$

The above equation clearly shows the essential of the 1:1 resonant orbit. The reason for the energy variation is the continuous work of the irregular gravitational field on the convected velocity. Figure 4 presents the energy variation corresponding to the 1:1 resonant orbit.



a) 1:1 resonant orbit in the inertial frame *IXYZ*

b) Corresponding orbital energy variation

Fig. 4 The 1:1 resonant orbits in *IXYZ* around the approximate model with u = 0.5 and k = 1 and its corresponding energy variations

Concluding Remarks

The resonant orbits in the vicinity of natural elongated bodies are investigated by using a unified approximate model of the rotating mass dipole. Taking the instantaneous orbital energy as a discriminator, four quadrants in the body-fixed frame are obtained according to the energy power. The orbital energy of the resonant orbit with its periapsis locating in quadrants II or IV increases whereas decreases for the orbit whose periapsis is in quadrant I or III. The reason for the energy variation of the resonant orbit is the continuous work of the irregular gravitational attracting on the convected velocity in the body-fixed frame.

Acknowledgements

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