James Clerk Maxwell (1831–1879), *Transactions of the Royal Society of Edinburgh*, Vol. 26, pp. 1-40

Miranda sectional diagrams courtesy N. Hammond

Miranda CSNB photomosaics transformed from PIA01490, NASA/JPL/USGS

Maxwell's Constant-scale Surfaces

Isolines Might Derive Unseen Districts

Beam Action as Precedent

Eisenlohr Projections

Historical Context:

Our emphasis

Isolines Derive Disruptive Networks

Maps as Maxwell Surfaces

Geological Boundary

Maxwell Surfaces

EXCERPTS FROM:

On Reciprocal Figures, Frames and Diagrams of Forces

A method for calculating stress in a structure **1869**

"Lastly, I shall extend the method to the investigation of the state of stress in a continuous body, and shall point out the nature of the function of stress . . . for stresses in two dimensions, extending the use of such functions to stresses in three dimensions.

F may be called the function of stress, because when it is known, the diagram of stress may be formed, and the components of stress calculated. ***The form of the function F is limited only by the conditions to be fulfilled at the bounding surface of the body.***

In cases of continuous stress . . . the symbolical method of calculation is still the best, although, as I have endeavoured to show, . . . analytical methods may be explained, illustrated, and extended by considerations derived from the graphic method."

In 1870, one year after Maxwell's force diagrams, Friedrich Eisenlohr of Bohemia presented the first and (until the introduction of world maps with constant-scale ***natural*** boundaries in 1993) the only projection with constant-scale edge.

Within a decade Maxwell was dead, and scientific exchanges between continental Europe and Britain had been minimal if not adversarial. He may not have known the possibility of testing his graphical method on unbounded finite surfaces embedded in a normal gravitational field.

With today's digital computers, stress analysis of complex surfaces in a superposed field is common—at least among architects and mathematicians—but the application to developed surfaces of icy satellites, as we try here, may be novel.

Tree of intrigue

Nodes of intrigue

Image Credit: Kelvin Song

Babylon

Gilgamesh

Low-resolution region that may be another ***confusion of tongues***, i.e., another Babylon

Ganymede Précis (Dione and Miranda Similar)

1: Draw a tree through centroids of old terrain

2: Make CSNB map from that tree ⟹ Map will feature new terrain

3: Derive the complementary tree ⟹ Nodes identify singularities and locales metrically correlated with old terrain

4: Make CSNB map from that tree ⟹ Map will feature old terrain

Typical Isoline

Typical Tree

Creusa

Cylindrical map courtesy T. Hare, from U.S. Geologic Survey Scientific Investigations Map 3237: Global geologic map of Ganymede, Collins, G.C., Patterson, G.W., Head, J.W., Pappalardo, R.T., Prockter, L.M., Lucchitta, B.K., and Kay, J.P., 2013

PIA18434
NASA/JPL-Caltech/Space Science Institute/Lunar and Planetary Institute

Miranda

Ganymede

Dione

Enceladus

100 miles

1000 miles

100 miles

Even with Gilgamesh at the periphery the metrical correlation to old terrain centroid is impeccable. See?

Metrical substitution obvious in CSNB map as undistorted in equal azimuth (central projection) as the only option in conventional projections.

moment

shear

telltale fissure

Compression

Tension

shear resolved to tension & compression

Characteristic shear cracks @ 45° to applied loads

Assuming Enceladan tiger stripes are shear cracks, gray arrows *(right)* show forces necessary to produce the observed orientation, if the Enceladan surface were a simply supported beam developed to the plane.

Enceladus, of course, is not a beam; it is a falling object subject only to parent-planet and sibling-moon gravitation.* Nonetheless, we test whether a suitably developed surface–a map with constant-scale ***critical*** boundary–may be a Maxwellian stress-analyzing surface. Results so far suggest yes.

Right: four deep-cusp Eisenlohr projections aligned with different tidal-force criticalities, A, B, C, and D.

Below and left: complementary constant-scale natural boundary maps.

Isolines act as wavefronts, and may resolve forces. Given any district and arrangement, isolines derive a centroidal network, a tree. Given any tree, isolines derive a consequent, maximally remote tree. Such metrically complementary trees may reveal causal relationships, especially on icy-shell satellites:

- At the point farthest from Ganymedian old terrain is Gilgamsesh.
- Along the tree most remote from Dionian "wisps" is Creusa.

LEADING

TRAILING

Deepening the cusp of the Eisenlohr unlike simultaneously preserving constant scale around the perimeter is mathematically quite challenging and likely would result in a definition of a projection that is not well known, transcendental functions expressed by a finite complex series.
–Dan Strebe, GEOART

Possible compositions for Enceladan tidal-stress-bounded Eisenlohr projections

A

B

C

D

Tidal Forces as Equivalent Concentrated Loads

Checks *(right)* indicate alignment of required shear vector with tidal vector ✓

$\mathbb{C}_N$ compression @ north pole

$\mathbb{C}_S$ compression @ south pole

$\mathbb{C}_T$ compression @ trailing hemisphere

$\mathbb{C}_L$ compression @ leading hemisphere

$\mathbb{T}_S$ tension toward stars

$\mathbb{T}_P$ tension toward Saturn

tension bulge

leading hemisphere

$\mathbb{C}_N$

ORBIT

$\mathbb{C}_T$

$\mathbb{T}_S$

$\mathbb{T}_P$

ORBIT

$\mathbb{C}_L$

$\mathbb{C}_S$

Side

top

(Left)
Drawn through old terrain, with tiger stripes at map middle
(blue network below)

(Right)
Drawn through prominent features, with tiger stripes at map extremities
(white network below)

(NASA/JPL-Caltech)

6 April 2014
apod.nasa.gov
Cassini Imaging Team/SSI/JPL/ESA/NASA

Background adapted from Rippled Rings: NASA/JPL/SSI, 2004

Enceladus geology map courtesy E. Crow-Willard & B. Pappalardo

Enceladus CSNB photomosaics transformed from PIA18434, NASA/JPL-Caltech/SSI/LPI

Poster & Handouts courtesy riot online imaging
Bill Mills
Image Wizard

* Also polar dichotomy and spin flattening.

Saturn adapted from PIA07268 An Infrared View of Saturn, NASA/JPL/STScI, 1998

Mapping and Graphic Stress Analysis for Icy Satellites Using Constant-Scale Natural Boundary Methods

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