TURBULENT CONCENTRATION OF MM-SIZE PARTICLES IN THE PROTOPLANETARY NEBULA: SCALE-DEPENDENT MULTIPLIER FUNCTIONS. J. N. Cuzzi¹, T. Hartlep², B. Weston³, and K. Shariff⁴, Jeffrey.Cuzzi@NASA.gov; ¹Ames Research Center, NASA, MS 245-3, Moffett Field, CA, USA; ²BAERI inc.; Sonoma, CA; ³U. C. Davis; Davis, CA.

Introduction: The initial accretion of primitive bodies (asteroids and TNOs) from freely-floating nebula particles remains problematic. Here we focus on the asteroids where constituent particle (read “chondrule”) sizes are observationally known; similar arguments will hold for TNOs, but the constituent particles in those regions will be smaller, or will be fluffy aggregates, and are unobserved. Traditional growth-by-sticking models encounter a formidable “meter-size barrier” [1] (or even a mm-cm-size barrier [2]) in turbulent nebulae, while non-turbulent nebulae form large asteroids too quickly to explain long spreads in formation times, or the dearth of melted asteroids [3]. Even if growth by sticking could somehow breach the meter size barrier, other obstacles are encountered through the 1-10km size range [4].

Another clue regarding planetesimal formation is an apparent 100km diameter peak in the pre-depletion, pre-erosion mass distribution of asteroids [5]; scenarios leading directly from independent nebula particulates to this size, which avoid the problematic m-km size range, could be called “leapfrog” scenarios [6-8]. The leapfrog scenario we have studied in detail involves formation of dense clumps of aerodynamically selected, typically mm-size particles in turbulence, which can under certain conditions shrink inexorably on 100-1000 orbit timescales and form 10-100km diameter sandpile planetesimals. The typical sizes of planetesimals and the rate of their formation [7,8] are determined by a statistical model with properties inferred from large numerical simulations of turbulence [9]. Nebula turbulence can be described by its Reynolds number $Re = (L/\eta)^{4/3}$, where $L = H\alpha^{1/2}$ is the largest eddy scale, $H$ is the nebula gas vertical scale height, and $\alpha$ the nebula turbulent viscosity parameter, and $\eta$ is the Kolmogorov or smallest scale in turbulence (typically about 1km), with eddy turnover time $t_\eta$. In the nebula, $Re$ is far larger than any numerical simulation can handle, so some physical model is needed to extend the results of numerical simulations to nebula conditions.

Cascade model and multiplier distributions: The spatial distribution of particle concentration can be captured statistically by a cascade model [7,9] which predicts the probability distribution functions (PDFs) for dense particle clumps; these PDFs are essentially volume fractions in the nebula having the necessary properties (solids mass and local vorticity) for planetesimal formation. A cascade model presumes that, as energy flows from large eddies to smaller ones, particles and fluid properties are partitioned unequally at each “level” of the cascade from “parent” into “daughter” eddies. The probability distribution functions (pdfs) of the so-called multipliers by which particle and fluid properties are partitioned have widths that are given by $1/\beta$; smaller $\beta$ means larger width, or a higher probability of strongly asymmetrical partitioning which, repeated over a number of levels, leads to clumpier or more intermittent local values of particle and fluid properties.

The wide range of scales between $L$ and $\eta$ is loosely called the inertial range; in this range the equations of motion are scale-free, and the multiplier pdfs for dissipation of turbulent kinetic energy $\epsilon$, which, like our preferred particles, is characterized by Kolmogorov eddy timescales, are known to be scale-independent in atmospheric flows across hundreds to thousands of Kolmogorov length scales [10; figure 1]. This fact motivated us to assume in [7] that the multiplier pdfs for particle concentration, which we denote by $\Phi$, were also independent of nebula turbulent eddy lengthscale. While this assumption was checked and seemingly validated (see figure 4 of [9]), we have always been concerned about it [7], and discrepancies between our model predictions and recent results of [11] have led us to determine more precisely the range of scales over which our assumption is or is not valid.

Here we report a scale dependence of the widths of the multiplier pdfs for both $\epsilon$ and $\Phi$, over a range of the largest scales, covering a decade or more between the energy-containing or integral scale $L$ and the top of a more rigorously scale-free inertial range. This scale dependence probably explains the differences between [7] and [11], and will affect the formation rate and masses of primary planetesimals in the scenario of [7,8].

Method and results: We obtained a complete dataset of particle trajectories in direct simulations covering 6 large eddy turnover times ($T_{eddy}$) in high $Re$ isotropic turbulence, for $2 = 6 \times 10^5$ “inertial” particles at each of a range of gas drag stopping times ($t_s > 0$), and for fluid tracers ($t_s = 0$) [12,13]. The original simulation was on a cube of 2048$^3\eta$ on a side, and had an integral scale of about 1024$\eta$. The data are thus similar to [9,11] but at a considerably higher $Re$, and showed a clear inertial range [12] extending over tens to nearly 1000$\eta$ based on the second order velocity structure function. The particle locations were spatially binned into snapshots in time, initially
Figure 2: The width of multiplier pdfs for particle concentration is the inverse of the β value, plotted here as a function of particle Stokes number \( St = t_o/t_s \). Each curve represents the situation at a different lengthscale (in units of \( \eta \)). Note that particles with \( St > 1 \) but still on the order of unity approach \( \beta \sim 5 - 10 \) by \( 64\eta \sim L/16 \) and generally stay there with the minimum \( \beta \) shifting slowly to smaller \( St \) at smaller scales and deepening towards the asymptotic value \( \beta \sim 3 \) (dashed line).

Figure 2 implies that, at least for the top decade or so of eddy lengthscales, multiplier pdfs are slightly narrower (larger \( \beta \)) than we had assumed and particles of \( St \sim 2 - 10 \) are more concentrated (ie, have broader multiplier pdfs than those for \( St = 1 \)). By contrast, particles of dm-size and larger have narrow multiplier pdfs at all but the very largest scales. Our preliminary results for \( \epsilon \) are similar to the particle results, in that the multiplier pdf which is so level-independent over a wide range of scales in atmospheric turbulence (figure 1) also starts out very narrow at the largest scales and only approaches \( \beta \sim 3 \) at \( 64\eta \) or so, roughly 1.5 decades below \( L \). We have not yet completed the correction of the \( \epsilon \) results for bias as of this writing.

Thus, the multiplier pdfs for both \( \epsilon \) and \( \Phi \) paint a consistent picture. In the top decade of lengthscales, turbulent stretching and vortex tube formation (see [8] and references therein) have not yet reached their fully developed state, so the statistics of \( \epsilon \) and \( \Phi \) of particles with Kolmogorov-eddy stopping times do not either. However, both \( \epsilon \) and \( \Phi \) multipliers approach the asymptotic value of \( \beta \sim 3 \) at slightly different but comparable scales of \( L/30 \), and we know from atmospheric turbulence that this asymptotic value then continues to hold over the entire inertial range, at least for \( \epsilon \). The important implication is that multiplier statistics are not characterized by their scale as a multiple of \( \eta \) but as a fraction of \( L \). Observed level independence of \( \epsilon \) to 3000\( \eta \) in atmospheric flows [10] is consistent with their much larger \( L \), such that 3000\( \eta \) < \( L/16 \).

The cascade model for planetesimal formation by turbulent concentration [10] can now be modified easily to allow for the observed level-dependence in the multipliers (figure 2), and since scales are resolved in the calculations reported here to almost the scale where level-independent pdfs are attained, future uncertainties in calculated IMFs for planetesimal formation will be greatly reduced. Moreover, particles that become most densely concentrated (are most intermittent, or have broader multiplier pdfs) in the range of eddy scales near the inertial scale have stopping times perhaps 2-10 times larger than we previously assumed, consistent with some previous work [11,14]. At smaller scales, \( St = 1 \) particles will increasingly dominate (figure 2). Because nebula properties such as turbulent intensity and gas density are not known to factors of order unity, these findings do not qualitatively change the outlook for the scenario (note the basic timescale revision found by [8]), but will lead to quantitative changes in predictions of planetesimal IMF and chordrule size distributions.