CONSTRAINING LONG-WAVELENGTH ELASTIC STRUCTURE OF THE LUNAR MANTLE USING GRAIL TIDAL LOVE NUMBERS. Chuan Qin, Shijie Zhong and John Wahr, Department of Physics, University of Colorado at Boulder, Boulder, Colorado 80309, USA (chuan.qin@colorado.edu).

Introduction: The Moon displays a number of long-wavelength or nearside-farside asymmetrical features. For example, 1) the farside topography is much higher than that of the nearside [1], 2) The KREEP terrane on the nearside has much higher concentration of incompatible and heat-producing elements than other areas [2], 3) the mare basalts erupted predominantly on the nearside from ~3.9 to ~3 Ga [3]. Moreover, deep moonquakes, detected by Apollo seismic stations, are located mostly on the nearside at depths of ~700 to 1200 km [4]. While the near-surface features suggest long-wavelength lateral heterogeneities in the early Moon's thermochemical structure, deep seismicity reflects the current state of the lunar interior and its largely nearside distribution suggests that the long-wavelength mantle structure may have survived to the present day [5].

The GRAIL (Gravity Recovery and Interior Laboratory) mission provides lunar gravity field data at unprecedented precision [6]. Recently, the JPL and GSFC analysis groups determined Love numbers at different spherical harmonics, i.e. k_{20}, k_{21}, k_{22}, and k_3, based on GRAIL Primary Mission measurements, and there are ~1% to 3% differences among k_2 values with uncertainties of ~0.1% to 1% [7, 8]. It is well known that for a spherically symmetric planet degree-2 Love numbers should be identical, while different k_2 values for different degree-2 harmonics may occur for laterally heterogeneous lunar structure due to mode-coupling effectt, according to our recent studies [9]. In this study, we seek to invert the GRAIL k_2 values for plausible 1-D and long-wavelength laterally varying (i.e., 3-D) structures of the Moon, using a perturbation method we newly developed [10].

Physical Model and Methods: The lunar tidal force is predominantly at spherical harmonics $(2,m_0)$ (i.e. l_0=2, m_0=0, 1, and 2, where l_0 and m_0 are spherical harmonic degree and order) [11]. When $(2,m_0)$ tidal force is applied to a spherically symmetric planetary body, the gravitational response of the body is at the same harmonic $(2,m_0)$ and can be described by Love number k_2 that is constant for different m_0. However, if laterally varying elastic structure (i.e., elastic moduli) exists in the planetary body, high-order tidal responses occur due to mode coupling effects in addition to regular response. According to our second-order perturbation theory, the high-order responses of a laterally varying planetary body to $(2,m_0)$ tidal forcing occur at $(2,m_0)$ (i.e., self-coupling mode) and other predictable harmonics. The amplitudes of different responses depend on both the pattern and the lateral variability of the structure. Considering three degree-2 tidal forcing components individually, the differences between the self-coupling responses of a given mantle structure lead to the differences in k_2 Love numbers.

We consider lateral heterogeneities in shear modulus μ and $\mu = \mu_0(r) + \delta\mu(\theta,\phi)$, where μ_0 is from 1-D lunar reference model and $\delta\mu$ denotes laterally varying structure. $\delta\mu$ can be expanded in terms of spherical harmonics as $\delta\mu = \sum_{l,m} \Delta_{lm} \cdot \mu_0 Y_{lm}(\theta,\phi)$, where Y_{lm} is the real-form spherical harmonic basis function and Δ_{lm} is the amplitude of lateral variability in μ at harmonic (l, m), respectively. We seek for the lunar mantle structure that can best fit the published k_2 values, by Monte Carlo sampling the elastic properties of the reference model and Δ_{lm} for each harmonic being considered. In this study, we only consider the longest wavelengths in our inverse models, i.e. l=1 and 2. For each set of sampled model parameters, we compute the tidal response of the Moon to $(2,m_0)$ forcing and especially k_2 Love numbers using our perturbation method. The lunar mantle structure is constrained by minimizing the misft between the calculated and published k_2 values.

Results and Discussions: In the current study, we choose the k_2 solutions with smaller uncertainties from the GSFC group, with $k_{20} = 0.024615 \pm 0.0000914$, $k_{21} = 0.023915 \pm 0.0000132$, and $k_{22} = 0.024852 \pm 0.0000167$. We consider two types of 1-D lunar reference models in our calculations, i.e. 1) Weber et al. model (M1) [12] and 2) a modified Weber et al. model (M2). Both M1 and M2 have a solid inner core, a fluid outer core, and 7 mantle and crust layers. Note that M1 model predicts k_2 that differs significantly from the averaged k_2 value ($k_{\_avg} = 0.024461 \pm 0.0000313$) (Fig.1A). The M2 model is obtained to fit the averaged k_2, by randomly varying the seismic velocities V_p and V_s of the 7 mantle and crust layers in M1 model by < 5%, while fixing all the other M1 parameters. We define χ^2–misfit to quantify the difference between k_2 from 1-D models and $k_{\_avg}$. 12 of 100,000 sampled models fit $k_{\_avg}$ with χ^2–misfit $< 10^{-6}$, and the M2 model is one of the 12 best fit models. For both M1 and M2 models, we introduce laterally varying structures at all possible degree-1 and degree-2 harmonics into the 7 mantle and crust layers with Δ_{lm} that are randomly generated

($-50\% \leq \Delta_{lm} \leq 50\%$) and run forward calculations to compute the k_2 Love numbers for (2,0), (2,1), and (2,2) tidal forces, respectively. χ^2–misfit is also defined for 3-D models as the difference between three modeled and GRAIL k_2 values. 3-D lunar mantle structure can be constrained by minimizing the misfit to the three k_2 Love numbers. We consider three different scenarios, depending on our inversion strategy: 1) (1,1) structure only, 2) all degree-1 structures, and 3) degree-1 and degree-2 structures.

Scenario I. Since a harmonic (1,1) structure can best represent the nearside-farside asymmetry, here we only introduce (1,1) structure into both M1 and M2 1-D models. For M1 model, the smallest misfit is ~4400 with Δ_{11}=±50%, while for M2 models, the smallest misfit is ~2300 with Δ_{11}=0% (Fig. 1A). The results demonstrate that (1,1) structure alone cannot account for the GRAIL k_2 solutions for either M1 or M2 model.

Scenario II. We consider all degree-1 structures, including (1,0), (1,1) and (1,-1) harmonics. For M1 model, the misfit reaches a minimum of ~1600 when Δ_{10}=±50%, Δ_{11}=±50%, and Δ_{1-1}=±18% (Fig. 1A). For M2 model, the smallest misfit of ~2300 requires Δ_{10}=Δ_{11}=Δ_{1-1}=0% (Fig. 1A). Note that for both scenarios I and II, there is no solution for M2 model. This is because all degree-1 structures tend to raise up k_2 Love numbers, but for M2 model, a reduced k_{21} is need to fit the data. However, with M1 model as 1-D reference, degree-1 structure can be inferred to reduce misfit to the k_2 Love numbers.

Scenario III. Here we include both degree-1 and degree-2 structures by adding (2,0), (2,1) and (2,2) harmonics in our models. The added degree-2 structures reduce the k_2 misfit significantly. For M1 model, the misfit drops to 0.12 with Δ_{10}=±40.2%, Δ_{11}=±48.9%, Δ_{1-1}=±18.5%, Δ_{20}=−12.2%, Δ_{21}=47.0%, and Δ_{22}=−48.9% (Fig. 1A and 1B). The best-fit k_2 values are k_{20} = 0.024592, k_{21} = 0.023917, and k_{22} = 0.024849. For M2 model, the minimum misfit is 120. The best-fit k_2 values are k_{20} = 0.024533, k_{21} = 0.024058, and k_{22} = 0.024882, with lateral variations of Δ_{10}=±1.3%, Δ_{11}=±4.6%, Δ_{1-1}=0.0%, Δ_{20}=−13.2%, Δ_{21}=0.4%, and Δ_{22}=−42.5%, predominated by degree-2 patterns (Fig. 1A and 1C).

We have demonstrated an efficient inversion approach based on a perturbation method to constrain the interior structure of the Moon using the reported GRAIL k_2 Love numbers. The reported differences among the k_2 Love numbers are unexpected for 1-D lunar mantle but may pose important constraints on 3-D lunar mantle structure. However, the inverted 3-D structure may depend on the assumed 1-D reference model (Fig. 1B and 1C). Love numbers at degree 3 may provide important additional constraints on the lunar mantle structure. We expect that improved k_2 and k_3 Love numbers solutions from GRAIL's Extended Mission should help greatly for our future modeling.

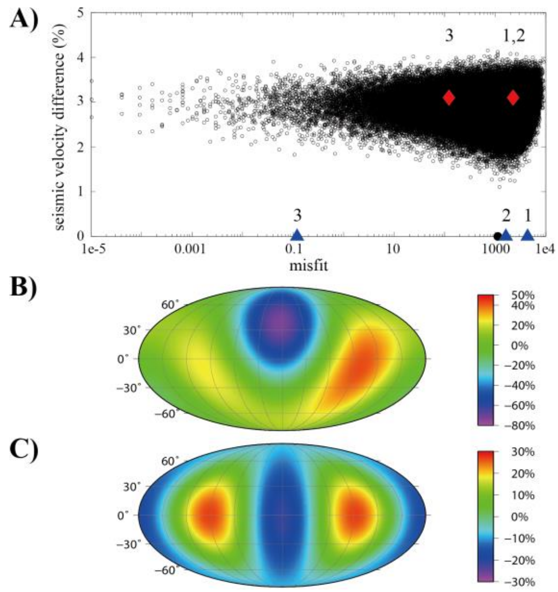

Figure 1. A) 1-D model difference relative to M1 model (2-norm of seismic velocity variations in percentage) vs. χ^2–misfit for both 1-D (relative to $k_{\_avg}$) and 3-D (relative to three GRAIL's k_2) cases. Black open circles are 100,000 randomly generated 1-D models against M1 model. Black closed circle represents the misfit in $k_{\_avg}$ for M1 model. Red diamonds are 3-D models from fitting k_2 for scenarios I, II, III, respectively, based on the chosen M2 model. Blue triangles show misfit for 3-D models from M1 model for scenarios I, II, III. B) Laterally varying (3-D) structure in shear modulus in scenario III that fit k_2 the best, given M1 model. C) Laterally varying (3-D) structure in shear modulus in scenario III that fit k_2 the best, given M2 model.

References: [1] Zuber et al. (1994) *Science* 266, 1839-1843. [2] Wieczorek and Phillips. (2000) *JGR* 105(E8), 20417–20430. [3] Wieczorek et al. (2006) *Rev. Miner. Geochem*. 60, 221-364. [4] Nakamura, Y. (2005) *JGR* 110, E0100. [5] Qin, C et al. (2012) *Icarus* 220, 100-105. [6] Zuber et al. (2008) *39th LPSC*, No 1391. [7] Lemoine et al. (2013) *JGR* 118, 1676-1698. [8] Konopliv et al. (2013) *JGR* 118, 1415-1434. [9] Zhong et al. (2012) *Geophys. Res. Lett.* vol 39, L15201. [10] Qin et al. (2013) *44th LPSC*, No 2459. [11] Wahr et al. (2009) *Icarus* 200, 188-206. [12] Weber et al. (2011) *Science* 331, 309-312.