GRAVITATIONAL POTENTIAL OF HAUMEA WITH A ROCKY CORE.

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Background: The Kuiper Belt Object (KBO) Haumea is a unique object in the solar system. Haumea apparently suffered a large impact billions of years ago: it is the only KBO associated with a collisional family, family members identified because they share icy surfaces with Haumea [1]. It is predicted [2] that some collisional family members (termed the "black sheep" of the collisional family) may be fragments of an undifferentiated rock/ice crust on the proto-Haumea. Modeling the structure of the proto-Haumea and testing this hypothesis requires knowledge of the current Haumea's core size.

The mass of Haumea is well constrained at 4.006×10^{21} kg because of its two satellites (Hi'iaka and Namaka) [3]. While its surface is icy [1], its overall bulk density is closer to that of rock. As determined using its light curve, Haumea also has an extremely fast rotation period of 3.9 hours (rotation rate $\omega = 4.5 \times 10^{-4}\,\mathrm{s}^{-1}$), which has deformed the body into a shape consistent with a Jacobi ellipsoid with semi-axes 980 x 759 x 498 km [5,6]. A Jacobi ellipsoid with these axis ratios is consistent with a homogeneous body with density $\approx 2585\,\mathrm{kg\,m^{-3}}$ [5]. With this average density, Haumea must have a rocky core under its icy exterior. Haumea therefore is not a homogeneous body.

All these factors complicate a determination of Haumea's internal structure and the size of its rocky core in particular. Assuming the mass and axis lengths above, a body of pure ice with density 1000 $\mathrm{kg\,m^{-3}}$ would have mass 1.552×10^{21} kg, implying a core mass $\approx 2.5 \times 10^{21}$ kg. But the size of this core depends on the material density. Its mean radius could vary from 565 km if its density were $\approx 3300\,\mathrm{kg\,m^{-3}}$, like that of ordinary chondrites [7], up to the full radius of the body, 718 km, if it were made of hydrated rock. The fast rotation of the body also means the core could possibly be a triaxial ellipsoid in shape, further complicating matters.

Our goal is to constrain the size and density of Haumea's core. We seek solutions of its density and individual axis lengths that yield the right mass of Haumea (assuming the remainder is ice), and which are consistent with the core's rotation rate ($= \omega$). In this abstract we describe preliminary steps toward this goal.

Jacobi Ellipsoids: We first assume that the core is a Jacobi ellipsoid with axis ratios fixed by its own density and rotation rate. That is, we ignore the gravitational effects on the core of the overlying ice. In that case [5],

$$\frac{2\omega^2}{3GM_\mathrm{c}} = \int_0^\infty \frac{u}{(a_\mathrm{c}^2+u)(b_\mathrm{c}^2+u)}\,du$$

and

$$a_\mathrm{c}^2\,b_\mathrm{c}^2 \int_0^\infty \frac{1}{(a_\mathrm{c}^2+u)(b_\mathrm{c}^2+u)\Delta(a_\mathrm{c},b_\mathrm{c},c_\mathrm{c})}\,du$$

$$= c_\mathrm{c}^2 \int_0^\infty \frac{1}{(c_\mathrm{c}^2+u)\Delta(a_\mathrm{c},b_\mathrm{c},c_\mathrm{c})}\,du,$$

where $\Delta(a_\mathrm{c},b_\mathrm{c},c_\mathrm{c}) = \left[(a_\mathrm{c}^2+u)(b_\mathrm{c}^2+u)(c_\mathrm{c}^2+u)\right]^{1/2}$ and the subscript refers to the core.

We simultaneously solve these formulas with a core mass $M_\mathrm{c} = (4\pi/3)\rho_\mathrm{c} a_\mathrm{c} b_\mathrm{c} c_\mathrm{c}$, where ρ_c is the assumed density of the core material. These provide 3 equations for the 3 unknowns a_c, b_c, c_c if ρ_c is treated as a free parameter. For example, if we were to fix $\rho_\mathrm{c} = 2.9\,\mathrm{g\,cm^{-3}}$, we would find $a_\mathrm{c} = 882$ km, $b_\mathrm{c} = 696$ km and $c_\mathrm{c} = 332$ km.

Constraining the fourth unknown, ρ_c, requires inclusion of knowledge about the icy mantle, which has known axes a, b and c from observations [5]. Similar expressions as the ones above would apply to the outer axes of Haumea except that it is not a homogeneous body. Because of this complication, no analytical solution probably exists to describe the equilibrium axis ratios as a function of the core mass. Instead we must try various core axis ratios and determine those that are consistent with Haumea's core mass and which also provide a gravitational field such that Haumea's outer surface is an equipotential. Only one value of ρ_c will do so.

Methods; At each fixed value of ρ_c, we fix a_c and use the integral equation above to solve for b_c as a function of a_c. The value of c_c is then fixed by the mass of the core. A solution for a_c is found that satisfies both integral equations.

We then determine the density ρ_c by treating it as a free parameter and varying it to minimize the following metric:

$$\mathcal{M} = \frac{1}{N}\sum_i^N \hat{n}_g \cdot \hat{n},$$

where the sum is taken over N points on the surface of the ellipsoid, $\hat{n}$ is the unit vector normal to the outer (surface) ellipsoid defined by Haumea's light curve, and $\hat{n}_g$ is the normal vector in the direction of the effective gravitational field, the gravitational field to which the centrifugal force $\omega^2 r$ has been added. If the surface ellipsoid is also an equipotential surface, then $\mathcal{M} = 1$ and the solution is optimal. We generate points on the surface ellipsoid by generating values of μ and ν in the ranges $-c^2 < \mu < -b^2$ and $-b^2 < \nu < -a^2$, thus generating points $x^2 = a^2(a^2+\mu)(a^2+\nu)/(a^2-b^2)/(a^2-c^2)$, with similar formulas for y and z. The normal is simply found: $n_x = 2x/a^2$, etc. The gravitational field is found by direct integration of the gravitational acceleration across grid elements.

Results: We expect the metric $\mathcal{M} = 1$ for spherical (non-rotating) bodies regardless of their density. We also expect $\mathcal{M} = 1$ for a homogenous Jacobi ellipsoid with the right axis ratios for a given uniform density. We expect $\mathcal{M} < 1$ in general for Haumea for a rocky core under an icy mantle, except for a single value of the core density ρ_c. Complete results regarding the internal structure of Haumea will be presented at the conference.

References: [1] M Brown et al. 2006 ApJ 639, 43. [2] JC Cook, SJ Desch & ME Rubin 2011 LPSC 42, 2503. [3] D Ragozzine & ME Brown 2009 AJ 127, 4766. [4] K Barkume, M Brown & E Schaller 2006 ApJ 640, 87. [5] P Lacerda & D Jewitt 2007 AJ 133, 1393. [6] L Lellouch et al. 2010 A&A 518, 147. [7] G Consolmagno et al. 2006 MAPS 41, 331.