A COMBINED COMPUTATIONAL/THEORETICAL APPROACH TO EXTENDING IMPACT SCALING FORMULAS.

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Introduction: The use of scaling laws via the Pi Theorem[1] for impact phenomena has been used for many years (see Ref. [2-6] and references therein), but the application of this technique is often limited to regime specific cases and only a single dimensionless parameter. While advantageous in terms of simplicity, this limits the fundamental power of the technique by excluding more interesting regions of parameter space and constraining the accuracy of the predictions obtained. Here we explore an extension of this technique to multiple dimensionless paremeters and higher orders in those parameters. To establish values for the undetermined coefficients in the theory we use results from the adaptive SPH code Spheral++, focusing on impacts into granite and copper.

Idealized Impacts and Scaling Laws: The idealized impact scenario we consider is shown in Fig. 1, with the parameters under consideration described in the figure caption. A fundamental assumption of this approach is that the relevant combination of impactor parameters for the scaling laws is the parameter *A* shown in the figure, with the material-dependent coefficient ε determining the degree to which the impact is kinetic-energy or momentum dominated.

The result of the impact is parameterized in the crater size *R* and the ejecta profiles *u(x)* and *r(x)*, where we make use of the assumption discussed in Ref. [4] that states that ejecta with a similar launch position *x* have similar characteristics. The relevant dimensionless parameters governing the values of *R*, *u*, and *r* can be simplified into the forms ga/v^2, ρ/δ, and $Y/\rho v^2$, as shown in Ref. [2-6].

At this point in the analysis the problem is typically simplified into being either in the gravity-regime (thereby eliminating the parameter $Y/\rho v^2$) or the strength-regime (eliminating ga/v). While many impacts of interest do indeed fall into one of these categories, there are possible impacts where both of these paremeters will be nearly equal in importance. Furthermore, if one wished for a more detailed examination of an impact in the strength regime the leading-order corrections would contain terms dependent on the gravity parameter, and vice versa for impacts in the gravity regime. Thus a more complete treatment is desirable.

To extend the scaling laws in a more complete and fully consistent manner we make use of the concept of an *effective theory*. Using the dimensionless paraemters generated by use of the Pi theorem as above, the effective theory approach then applies a completely general expansion of the arbitrary function of those parameters:

$$\begin{aligned}\pi_p &= F(\pi_1, \pi_2, \ldots)\\ &= \prod_{i=1}^{k} \sum_{j=-\infty}^{\infty} c_{ij}\pi_i^j\end{aligned}$$

where π_p is the dimensionless parameter that contains our target variable (e.g. crater radius *R*), the c_{ij} are undetermined coefficients, and the π_i are the dimensionless numbers from the Pi theorem. So long as the expansion is completely general and is convergent to the function *F* over the domain of interest this expansion will give the correct result.

The functional form of this expansion is then limited by consideration of the relevant limits of the function. For applications to cratering an example of this would include the fact that an infinitely strong medium (i.e. *Y* becomes very large) the crater radius will go to zero. This would typically restrict the allowed values of *j* in the above expansion to be either definite positive or definite negative, depending on the specific contruction of the expansion. Note that the standard expansion utilized so successfully in Ref. [2-6] terminates after the leading contribution and assumes that some subset of the π_i are negligible, but that such a limit is recoverable from this treatment. Furthermore, the standard empirical treatment for connecting the strength and gravity regimes falls naturally out of this process.

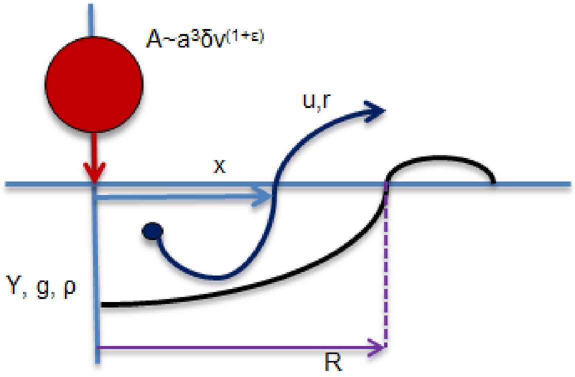

Figure 1: Setup of the idealized hypervelocity impact problem. The impactor of radius *a*, density *δ*, and velocity *v* is incident on an infinite plane of material with strength *Y*, density *ρ*, and in a gravitational field with acceleration *g*. A crater of radius *R* will be formed with ejecta being launched from various positions *x* with velocity *u(x)* and ejecta radius *r(x)*.

Impact Simulation Contributions: Upon limiting the form of the expansion one is left with many undetermined coefficients to be set via experiment or simulation. In order to model the late time evolution of these scenarios we emply the ASPH (Adapative Smoothed Particle Hydrodynamics [9,10]) code Spheral++, which follows the evolution of the hydrodynamics with strength and porosity, gravity, damage evolution, fracture and failure. Specifically we use Spheral++ with the Livermore EOS tables, a Steinberg-Guinan model for material strength, and the Grady-Kipp-Benz-Asphaug damage model for sub-scale material failure.

To validate the Spheral runs we compared the crater size and ejecta information to an experiment using a 1.26 cm aluminum impactor incident at 7 km/s on a granite surface[9], and compared to the crater volume from a 3 mm stainless steel impactor incident at 1 km/s[10]. Both crater volumes were correct to within 5% and the resulting comparison of ejecta fragment sizes from granite is shown in Fig. 2.

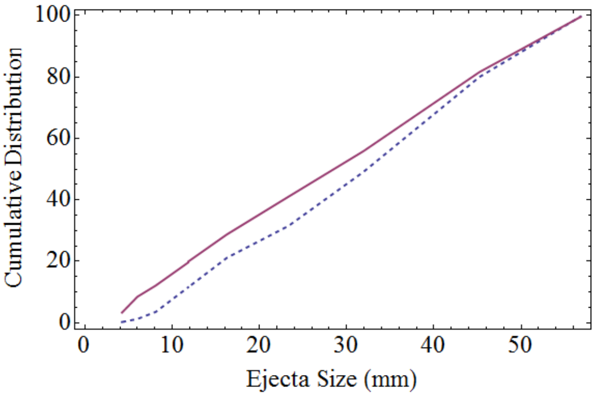

Figure 2: Fragment size distribution (using the Sauter mean diameter) from an impact into granite from experiment (solid, red) and Spheral simulation (dashed, blue).

An example of the late-time phase of an impact event using Spheral is shown in Fig. 3. Here a stainless steel impactor is incident on a copper target, with the resulting damage and fracture of both the impactor and the target region. By running many of these simulations at varying impactor velocities, sizes, and gravity values a much larger portion of parameter space can be explored than is available through experiment or planetary observation. As such, we can access the crossover region between the strength regime and the gravity regime for a detailed treatment. This will also fix the values of the coefficients of the higher order terms in the expansion, leading to a greater understanding of higher order effects in the scaling laws. In addition, the theoretical understanding from this method sheds light on several assumptions and approximations used by the scaling law community, specifically the degree to which ejecta velocity profiles are independent of strength and gravity regime considerations for ejecta sufficiently far from the crater edge.

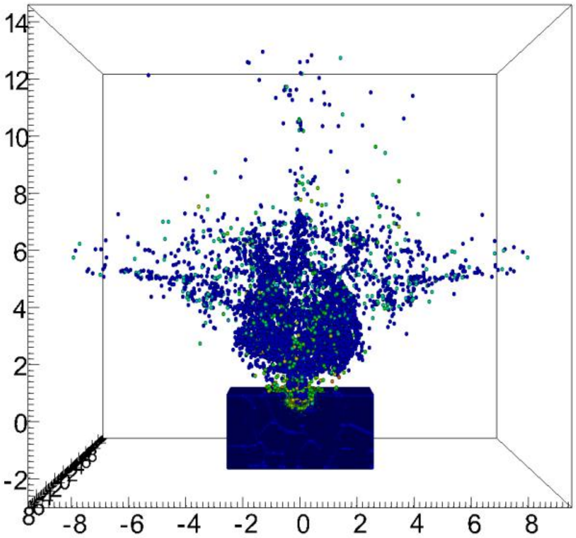

Figure 3: Results from a Spheral simulation of a stainless steel impactor incident at ~1 km/s into a copper slab, at 20 μs past impact. The axes dimensions are in cm and the individual Spheral nodes are color coded by effective damage, from blue (nearly undamaged) to green (slight damge) to red (dust).

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