SURFACE STABILITY OF RAPIDLY SPINNING SPHEROIDS. D.J. Scheeres, *U. Colorado, Boulder (scheeres@colorado.edu)*, P. Sánchez, *U. Colorado, Boulder*.

We study the surface slopes of regolith on spinning, gravitating bodies to identify how they evolve across the surface of these bodies, the direction of failure for slopes that exceed the angle of repose, and an analysis of how the shape of the body may be modified based on these results. We find specific characteristics for body surfaces when spun to or near disruption and compare them with the observed characteristics of the class of small, oblate and rapidly spinning asteroids such as 1999 KW4 Alpha, Bennu and 2008 EV5.

Introduction The angle of repose and conditions for global landslides on the surfaces of small, rapidly spinning, spheroidal asteroids are studied. Applying techniques of soil mechanics, we develop a theory for, and examples of, how regolith will fail and flow in this microgravity environment. Our motivation is to develop an understanding of the "top-shaped" class of asteroids based on analytical soil mechanics. Our analysis transforms the entire asteroid surface into a local frame where we can model it as a conventional granular pile with a surface slope, acceleration and height variations as a function of the body's spin rate, shape and density. A general finding is that the lowest point on a rapidly spinning spheroid is at the equator with the effective height of surface material monotonically increasing towards the polar regions, where the effective height can be larger than the physical radius of the body.

We study the failure conditions of both cohesionless and cohesive regolith, and develop specific predictions of the surface profile as a function of the regolith angle of friction and the maximum spin rate experienced by the body. The theory also provides simple guidelines on what shape a body may take, although we do not analyze gravitationally self-consistent evolution of the body shape. The theory is tested with soft-sphere discrete element method granular mechanics simulations to better understand the dynamical aspects of global asteroid landslides. We find significant differences between failure conditions for cohesive and cohesionless regolith. In the case of cohesive regolith, we show that extremely small values of strength (much less than that found in lunar regolith) can stabilize a surface even at very rapid spin rates. Cohesionless surfaces, as expected, fail whenever their surface slopes exceed the angle of friction. Based on our analysis we propose that global landslides on spheroidal bodies will generally result in the flow of material towards the equator.

Surface Slope and Accelerations Assume the surface of a sphere of radius R is covered with regolith of a depth H where we implicitly assume that $H \ll R$. The total mass of the body is M and is assumed to have a constant density. The regolith has mechanical properties of a soil, with a friction angle ϕ and a cohesion c. We study the stability of the surface of this sphere as its spin rate is increased under some slow, exogenous effect such as YORP. With this model the surface accelerations are characterized by the total gravitational parameter of the body, $\mathcal{G}M$, and the spin rate of the sphere, ω. At any point on the surface of the body the total acceleration acting on the regolith is comprised of a radial and transverse acceleration:

$$\boldsymbol{a}_n = g_0 \left[-1 + \tilde{\omega}^2 \cos^2 \delta\right] \hat{\boldsymbol{r}}$$

$$\boldsymbol{a}_t = -g_0 \tilde{\omega}^2 \cos\delta \sin\delta \hat{\boldsymbol{t}}$$

$$a = g_0 \sqrt{1 - (2 - \tilde{\omega}^2) \cos^2 \delta \tilde{\omega}^2}$$

where δ is the latitude, $g_0 = \mathcal{G}M/R^2$ is the surface gravity, $\hat{\boldsymbol{r}}$ is the radial unit vector, $\hat{\boldsymbol{t}}$ points towards the pole of the spinning body and away from the equator, and $\tilde{\omega} = \omega\sqrt{R/g_0}$. If $\tilde{\omega} < 1$ then the entire surface is spinning less than the surface disruption limit.

Using the accelerations, we can define the local slope on the surface of our spinning sphere. The tangent of the slope equals the ratio of the tangential and normal accelerations.

$$\tan\theta = \frac{\tilde{\omega}^2 \cos\delta \sin\delta}{1 - \tilde{\omega}^2 \cos^2 \delta}$$

See Fig. 1 for a representation of slope as a function of latitude and spin rate. Thus we note that the slope is always zero at the equator and the pole (for $\tilde{\omega} < 1$). Further, the slope is always positive leading away from the equator, meaning that the equator of a spinning sphere is the minimum point in the surface geopotential. The maximum value of slope as a function of $\tilde{\omega}$ is found to be:

$$\tan\theta^* = \frac{\tilde{\omega}^2}{2\sqrt{1 - \tilde{\omega}^2}}$$

which occurs at a latitude defined by $\tan\delta^* = \sqrt{1 - \tilde{\omega}^2}$. Thus the latitude of maximum slope occurs at 45° latitude for nearly zero spins and moves down towards the equator for more rapid spins. At the same time, the maximum value of the slope increases with increasing spin rate, until it reaches a value of 90° at the equator for $\tilde{\omega} = 1$. At extreme spin rates it is also instructive to note that the slope takes on a specific structure. Let $\tilde{\omega} \to 1$ to find $\tan\theta = \cot\delta$, or $\theta = \pi/2 - \delta$. Thus, even thought the slope goes to 90° at the equator, it is limited to the colatitude across the surface of the body. Specifically, in the region of the pole the slope will always be low and close to zero, independent of the spin rate of the body.

Ignoring varying gravity as a function of radius we can define an effective height in terms of slope alone by integrating the relation $dh = R\tan\theta d\delta$ to find:

$$h(\delta, \tilde{\omega}) = R \ln \sqrt{\frac{1 - \tilde{\omega}^2 \cos^2 \delta}{1 - \tilde{\omega}^2}}$$

See Fig. 2 for the effective height as a function of latitude and spin rate. The highest point is always at the pole and the lowest point at the equator. Thus we note that particle migration will always flow towards the equatorial regions. The total acceleration, however, is minimum at the equator and maximum at the poles. Thus we find the dichotomous situation where material flows from the region of highest effective gravity to the region of lowest effective gravity.

Cohesionless Surface Failure and Shapes It is of interest to study the failure of the surface of a rapidly spinning sphere once the angle of repose is violated. We develop an analytical theory that is applicable to the surface of a spherical body and apply it to predict the characteristics of the shape of a failed body. Once the angle of repose of the regolith is exceeded (equal to the angle of friction), we note that portions of the surface regolith will flow towards the equatorial region. In our simple, analytical model we assume that this migrated material will ultimately settle into a zero slope condition at the equator (note, this is a strong assumption and will be relaxed in future research). This is to be contrasted Minton [1] where surfaces were formed at a constant value of angle of repose.

Applying these simple failure rules we can predict the shapes of the surface of the spheroid as it undergoes failure at progressively higher spin rates. For a friction angle of 40° we find a striking similarity between the profile of 1999 KW4 Alpha and the ideal shapes predicted from the failure law (see Fig. 3). We note, however, that this particular asteroid shape may also have been strongly influenced by the formation of its binary companion. In particular, there are different theories for how such bulges form in the context of a binary system (see Walsh et al. [2], Jacobson and Scheeres [3] and Hirabayashi and Scheeres [4]). To properly identify how such characteristic ridges form requires the different competing theories to be fully explored and compared.

To further test these theories we are developing a simulation model of surface regolith failure. Figure 4 shows one particular attempt using a soft-sphere DEM code [5].

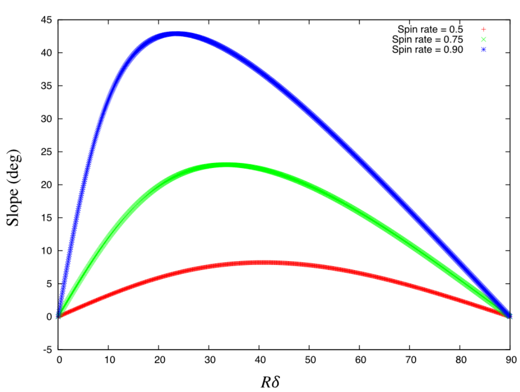

Figure 1: Slope as a function of latitude and spin rates.

References: [1]: Minton, Icarus 195: 698-704 (2008). [2]: Walsh et al., Nature 454: 188-191 (2008). [3]: Jacobson and Scheeres, Icarus 214: 161-178 (2011). [4]: Hirabayashi and Scheeres, LPI Abstract (2014). [5]: Sánchez and Scheeres, ApJ 727:120 (2011).

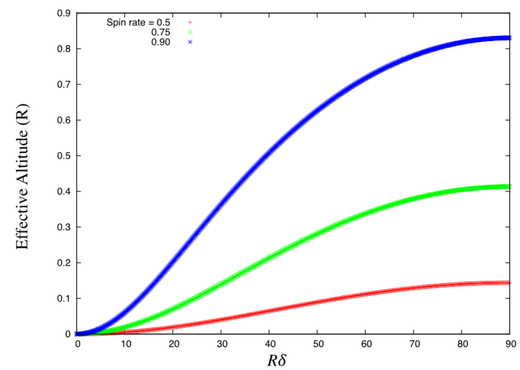

Figure 2: Effective height for latitude as spin rates.

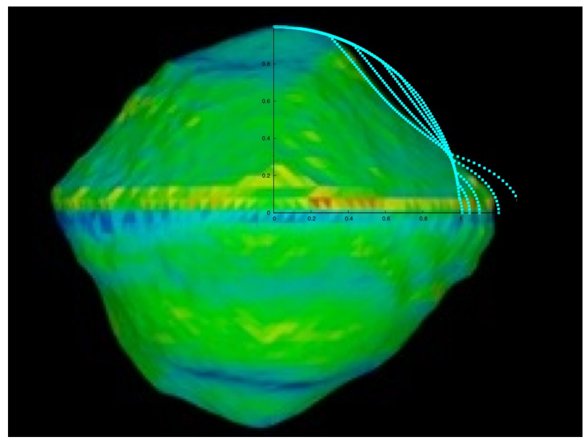

Figure 3: Comparison between 1999 KW4 Alpha and surface slope failure shapes for increasing spin rates.

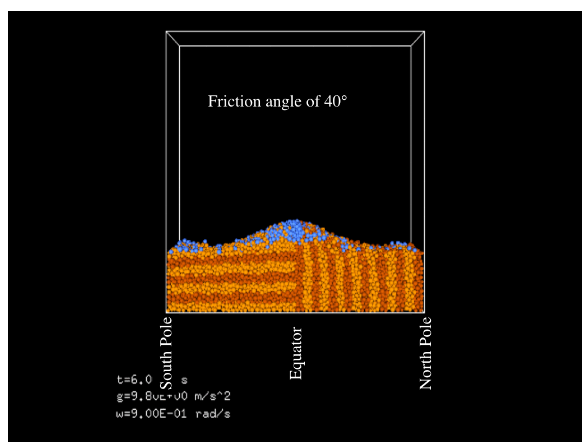

Figure 4: Result of a granular mechanics simulation mimicking the surface of a spheroid.