SOIL PERMEABILITY: ACCURACY OF THE KOZENY-CARMAN EQUATION IN SHALLOW FLOW PROBLEMS ON MARS AND EARTH. H. G. Sizemore. Planetary Science Institute (1700 East Fort Lowell, Suite 106, Tucson AZ, sizemore@psi.edu).

Introduction:

Quantifying the permeabilities of specific granular materials is an important step in understanding a variety of physical systems in which fluids move through soils and regoliths. In the current Martian climate, there is considerable interest in how transient or partial melting of shallow ground ice and flow of liquid water might contribute to gully-like features, recurring slope lineae (RSL) [1], and the development of massive or excess ground ice [2]. Permeability, *k*, is a key parameter in physical models of all these systems. Measuring and predicting permeability in porous materials is also an area of active research [3, 4, 5].

Typically, numerical models of shallow water flow on Mars have incorporated empirical measurements of *k* [e.g. 1, 2], as have numerical models of terrestrial frost heave [e.g. 6, 7]. However, predictive calculations of *k* are possible based on soil properties that are more readily measured in the laboratory and by landed spacecraft, such as porosity and specific surface area. Saruya et al. [8] employed a form of the Kozeny-Carman (KC) equation to predict *k* in their analysis of laboratory ice lens growth. Chapuis and Aubertin [9] reviewed a broad range of empirical measurements of *k* in natural soils. They concluded that the standard form of the KC equation can predict hydraulic conductivity and permeability to within a factor of a few in natural sands and many clays, *if* an accurate measurement of specific surface area is available. However, theoretical studies [e.g. 3, 4] have suggested that the KC equation should not be applied predictively and Hillel [10] cautioned against its use.

Here, I make a systematic comparison between empirical *k* values measured in soils with direct relevance to Martian flow problems, and values predicted by the KC equation. I discuss the relative accuracy of the different forms of the equation, and implications for the use of calculated and empirical values of *k* in physical models of shallow flow in planetary environments.

Methods:

Forms of the Kozeny-Carman Equation. A variety of forms of the KC equation appear in the geotechnical, materials physics, and planetary science literature. I assessed the efficacy of five of these. Chapuis and Aubertin [9] summarized the development of the standard form of the equation:

$$k = \frac{\phi^3}{cS^2(1-\phi)^2} \tag{1}$$

where k is the permeability [m^2], ϕ is porosity, c is a unitless constant commonly taken to equal 5, and S [m^{-1}] is the specific surface, (ρ_{bulk} SSA). Equation (1) is also the form discussed by Hillel [10]. I evaluated four additional forms that include a specific surface term.

From [3]:

$$k = \frac{(1-\phi)^3}{c_o S^2} \tag{2}$$

where c_o = 2 is a constant. And from [5]:

$$k = \frac{\phi^3}{c\tau^2 S^2} \tag{3}$$

$$k = 10^{2\phi}\frac{\phi^3}{c\tau^2 S^2} \tag{4}$$

$$k = 15\frac{\phi^4}{c\tau^2 S^2} \tag{5}$$

where c = 5 and τ is tortuosity.

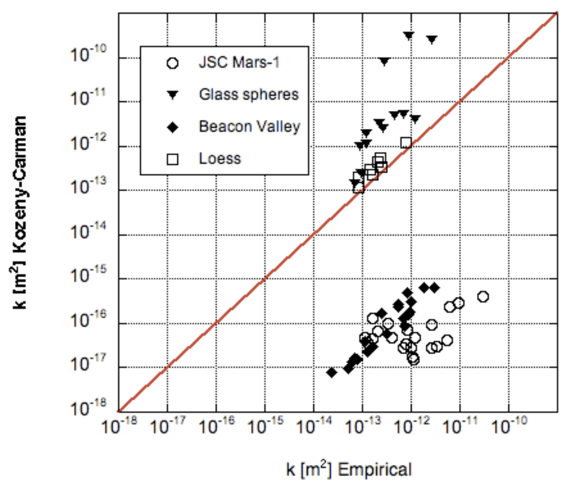

Figure 1. Comparison of *k* calculated via equation (1) to empirical values from [12] for four groups of Mars-analog soils. Note that particle size distribution, porosity, and tortuosity vary within each soil type.

Data sets. I relied primarily on a soil-characterization data set from [11], which includes measurements of intrinsic and bulk density, tortuosity, permeability, and BET surface area for five groups of Mars-analog soils. Because this data set is comprehensive, I was able to make direct comparisons between theoretical values of *k* and empirical values measured in the same soil sample via a dry Argon flow technique (Figs. 1 & 2). I also used a less complete data set [12], which included empirical *k* values measured by standard (wet) permeameter techniques for two soils and specific surface areas for > 20 soils of interest in terrestrial frost heave studies. The second data set allowed me to evaluate the performance of the KC equation in making order-of-magnitude estimates of *k* over a broad range of *S* using standard assumptions for porosity and intrinsic density.

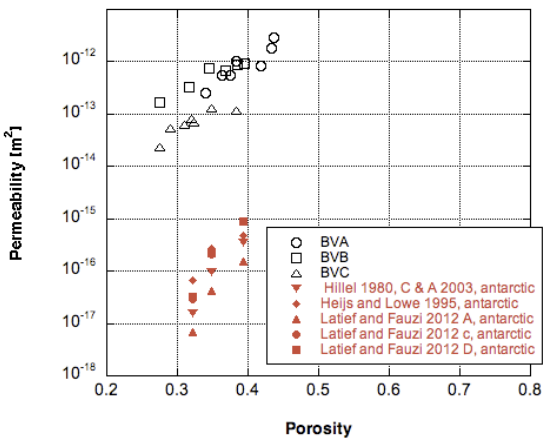

Figure 2. Comparison of empirical *k* (dry Argon technique) to *k* calculated via equations (1)-(5) for soils sample at three locations in Beacon Valley, Antarctica.

Results and Discussion: As shown in Fig. 1, the performance of the standard KC equation was highly variable between soil types. The equation predicted *k* in Fox Permafrost Tunnel loess with reasonable accuracy. It overestimated *k* for glass spheres with diameters > 20 µm by an order of magnitude or less. It overestimated *k* for glass spheres with diameters < 20 µm by more than two orders of magnitude. Equation (1) *under*estimates *k* in the Antarctic soils by three orders of magnitude and underestimates *k* in JSC Mars-1 by three to five orders of magnitude depending on grain-size component.

Although I observed differences in the performance of Equations (1)-(5), no form performed systematically better or worse across a variety of soil types. This is noteworthy, given that equations (3)-(5) were developed for cohesive sandstones, and were expected to perform less well in unconsolidated soils.

Chapuis and Aubertin [9] suggested that soils with bi- or tri-modal pore size distributions might not exhibit KC type behavior, which might explain the poor performance of equation (1) for small diameter glass spheres (bi-modal pore size distribution) and JSC Mars-1 (tri-modal). However, equation (1) accurately predicts *k* in the Fox Permafrost Tunnel loess (distinctly tri-modal), and grossly underestimates *k* in the Antarctic soils (narrowly peaked).

Chapuis and Aubertin [9] also suggested that disagreement between equation (1) and empirically measured *k* may indicate a problem with experimental technique. This is worth considering, given that hydraulic conductivity, not permeability, was their focus. Likewise, flow of liquid water is the focus of many physical models that apply empirical *k* to Mars [e.g. 1, 2], but the most comprehensive set of *k* measurements in Mars analog soils was produced using a dry technique [11].

The KC equation is not as broadly applicable or accurate as indicated by [9] for three reasons. First, the equation grossly underestimates Argon-based *k* values in the sandy Antarctic soils. These soils do not have a significant clay component that could contribute to major differences between wet and dry measurement techniques. Second, the basic form of all five equations I evaluated produces unrealistically low values of *k* for specific surface areas greater than a few 10s of m^2/g with otherwise reasonable assumptions of bulk density, tortuosity, etc. Third, the KC equation grossly underestimates *k* measured by wet techniques for two soils in the second data set I considered [12].

Frost heave and flow rates in physical models are proportional to *k*. Uncertainty at a 2-3 order of magnitude level could affect the interpretation of model results significantly. Given its poor performance in standard Mars analog soils *and* glass spheres, no form of the KC equation should be relied on as a predictive tool. However, there may still be a need for new measurements of *k* in a variety of soils via wet permeameter techniques.

References:

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