EVOLUTION OF THE PROTOLUNAR DISK**.** Wm. R. Ward, Planetary Science Directorate, Southwest Research Institute, 1050 Walnut St., Boulder, CO 80302, ward@boulder.swri.edu

Introduction: The giant impact (GI) hypothesis of lunar formation proposes a ~Mars-sized impactor obliquely collided with the Earth during its final stage of accretion and launched material into orbit around the Earth from which the Moon subsequently formed [1,2]. Following its emplacement, disk material interior to the Roche distance, $r_R \sim 3R$ (where $R = 6.38\times10^8\,cm$ is the Earth's radius), cannot undergo accretion due to tidal shear. Instead, material will viscously spread, with some being re-accreted by the Earth while the rest diffuses outward. Material flowing across the Roche boundary can be incorporated into accreting moonlets, which eventually coalesce into the Moon itself. The diffusion rate depends on the strength of the disk viscosity, $\nu \sim \pi^2G^2\sigma^2/\Omega^3$, due to gravitational instability [2-4], where σ is the disk surface density and Ω its orbit frequency. Early estimates of the spreading timescale for the disk were very short due to the assumption that disk material nearly fully was condensed. This assumption proved unjustified because the viscous dissipation rate, $\dot{E}$ = $(9/4)\sigma\nu\Omega$, would be so fast that a significant portion of disk material would vaporize [5]. If *both* vapor and condensed silicate are present, a stratified structure could develop wherein a gravitationally unstable, midplane magma layer, $\sigma(r)$, is surrounded by a stable vapor layer, σ_V [6]. The relative proportions of the two phases can then adjust so that the combined amount of material diffusing out of the magma layer to be accreted by the Earth or Moon matches the amount of condensing vapor raining down on the magma layer. The condensation rate is given by the amount of released latent heat, $l = 1.7\times10^{11}$ *ergs/gm*, required to make up the deficit between the photospheric radiation, $\mathcal{L} = 2\sigma_{SB}T^4_{ph}$ from the disk surfaces, with σ_{SB} being the Stephan-Boltzman constant and T_{ph} = 2000°K the phase equilibrium temperature and the viscous energy dissipation, *i.e.*,

$$-\partial\sigma_V / \partial t = (\mathcal{L} - \dot{E}) / l \quad . \qquad (1)$$

Freely Expanding Disks: The resulting magma layer's evolution is characterized by a varying in-plane radial flux of material, $F(r, t)$, that can be found from Euler's equation, $F = -\partial g/\partial h$, where $g = 3\pi\sigma\nu h$ is the viscous couple, and $h = (GMr)^{1/2}$ is the specific angular momentum at radius r in a Keplerian disk with $M = 5.97\times10^{27}\,g$ being the Earth's mass. This is differentiated to find $\partial^2g/\partial h^2 = -\partial F/\partial h$ and is to be combined with the continuity equation, $\partial(\sigma+\sigma_V)/\partial t = -(2\pi r)^{-1}\partial F/\partial r = -(G^2M^2/4\pi h^3)\partial F/\partial h$. (2)

The LHS of eqn.(2) can be expressed in terms of g and h by using eqn. (1) together with the substitutions,

$$\Omega = (GM)^2/h^3 \;\; ; \;\; \sigma = (GM)^2g^{1/3}/\pi(3G^2)^{1/3}h^{10/3} \; . \; (3)$$

Differentiating σ with respect to time ultimately leads to the diffusion equation

$$\frac{4}{3}\left(\frac{g}{3G^2h}\right)^{1/3}\frac{1}{g}\frac{\partial g}{\partial t} = \frac{\partial^2 g}{\partial h^2} + \left(\frac{4\pi\mathcal{L}}{G^2M^2}h^3 - 3G^2M^2\frac{g}{h^4}\right)\frac{1}{\ell} \qquad (4)$$

with the final source terms on the RHS due to the material condensing from the vapor reservoir.

Quasi Steady-state Solutions: The active supply enables the magma layer to maintain a quasi steady-state where $\partial g / \partial t \approx 0$. The layer tends to be driven to this state from its initial post-impact configuration. Of course, eqn. (1) tells us that this is not the case for the vapor layer. To solve for the magma steady-state profile, it is convenient to cast the defining equation in non-dimensional form by introducing the following forms $x' \equiv x / x_o$ for each variable x, where their reference values are defined as:

$$h_o \equiv (GMR)^{1/2} = 5.05\times10^{14}\,cm^2 / s$$

$$g_o \equiv (4\pi\mathcal{L}R^4 / h_p)(GM / lR) = 2.76\times10^{31}\,g\cdot cm^2 / s^2$$

$$\sigma_o \equiv (M / \pi R^2)(g_oR / 3GM^2)^{1/3} = 6.30\times10^6\,g / cm^2$$

$$\nu_o \equiv h_p(g_oR / 3GM^2)^{2/3} = 9.20\times10^8\,cm^2 / s$$

$$F_o \equiv g_o / h_p = 5.47\times10^{16}\,g / s \, .$$

and are evaluated for the Earth.

The equation giving the steady state of g takes the compact form,

$$d^2g' / dh' - \eta^2 g' / h'^4 = -h'^3 . \qquad (5)$$

which represents a one-parameter set of curves where the parameter $\eta^2 \equiv 3GM / lR$ is proportional to the ratio of the specific potential energy at the planet's surface to the latent heat of vaporization. Eqn. (5) can be solved analytically in terms of modified Bessel functions, and its solution for the Earth value of $\eta = 3.33$ is displayed as the blue curve(s) in Fig. 1 versus $r' \equiv r / R = h'^2$. The other normalized variables are then found from the relationships: $\sigma' = g'^{1/3} / h'^{10/3}$, $\nu' = g'^{2/3}\,h'^{7/3}$, and $F' = -\partial g' / \partial h'$.

Also shown in Fig. 1 are (*i*) the solution for the case $\eta = 0$ (grey curves), appropriate when the viscous heating is negligible, $\dot{E}/\mathcal{L} \to 0$, and (*ii*) the surface density (dashed curve in top panel) for which viscous dissipation could supply all the radiated energy, $\dot{E}/\mathcal{L} \to 1$. The proto-lunar disk is much closer to (*i*), implying viscous heating plays a relatively minor role, although this would not necessarily be true for other planet/disk situations [7].

There is a stagnation point in the flux, $F' = 0$, near the halfway mark, with a positive value exterior to that indicating material flowing toward the accretion zone of the Moon. Negative values of F' in the inner portion of the disk indicate material flowing toward and destined to be accreted by the Earth. The flux at the earth boundary and at the Roche distance are $F_p' = -0.610$ and $F_R' = 1.129$. Thus, the total rate of magma leaving the disk through both boundaries is $(F_R' - F_p')F_o = 9.51\times10^{16}\ g/s = 0.0408\ m_{moon}/yr$. By contrast, the mass of the magma layer is maintained at

$$m_{mag} = 2\pi\int \sigma r dr = 4\pi\Sigma R^2 \int_1^{\sqrt{3}} (g'/h')^{1/3} dh' = 0.138 m_{moon} . \qquad (6)$$

Consequently, it takes 3.4 yrs to completely replenish a magma layer. However at the beginning, most of the disk material is in its vapor state surrounding the magma layer and continues to condense, so it requires $m_{disk}/m_{magma} \sim 7.2(m_{disk}/m_{moon})$ "cycles" to deplete the disk. Assuming a total vapor + magma mass of m_{disk} ~2 lunar masses, the time to deplete the disk at this rate would be ~ 50 years. This is roughly the time scale for the disk to radiate all of the latent heat released by condensation at the two ~2000^o*K* photospheric surfaces [5].

Summary:

- The overall timescale for the protolunar disk evolution is set by the time to radiate its latent heat as suggested by Thompson & Stevenson [5].
- If it has a stratified structure [6], the magma layer spreads on a time scale of a few years as suggested by Ward & Cameron [2]. However, a quasi steady-state magma layer contains only ~14% of a lunar mass, so most of the disk material starts out in the vapor reservoir.
- A freely expanding disk could supply material to the accretion zone at the rate of F_p ~0.027 lunar masses/yr, which is ~65% of the total material diffusing out of the disk (the rest being accreted by the Earth).
- Finally, this free expansion process could be curtailed if an accreting Moon begins to tidally confine the disk [8] and lead to a hiatus of reduced flow across the Roche boundary until the lunar embryo tidally migrates beyond its 2:1 resonance with the Roche boundary. [7, 8].

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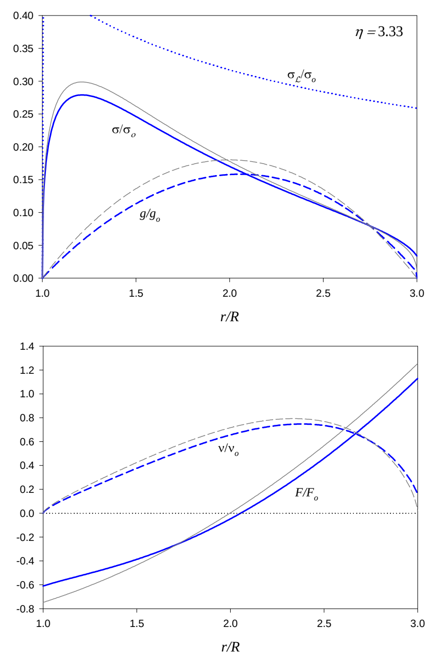

Fig. 1. The profile of a quasi steady-state protolunar disk undergoing free expansion. (top) The normalized surface density and viscous couple, (bottom) the normalized viscosity and in-plane flux. The case for the protolunar disk with $\eta = 3.33$ is shown in blue; the case for no viscous heating, η = 0 (grey curves) is shown for comparison. Also shown (top, dashed line) is the surface density, $\sigma_{\mathcal{L}}$, for which the viscous heating in the magma layer would completely furnish the energy radiated from the surfaces of the disk.