Automated Dynamic Modelling Of Fireballs For The Australian Desert Fireball Network. E. K. Sansom[1], P. A. Bland[2] and J. Paxman[3], [1]Department of Applied Geology, Curtin University, GPO Box U1987, Perth, WA 6845, Australia, eleanor.sansom@curtin.edu.au, [2]Department of Applied Geology, Curtin University, GPO Box U1987, Perth, WA 6845, Australia, p.a.bland@curtin.edu.au, [3]Department of Mechanical Engineering, Curtin University, GPO Box U1987, Perth, WA 6845, Australia, j.paxman@curtin.edu.au.

Introduction: Fireball observations from five dedicated camera networks have led to the recovery of 10 meteorites. Although this is a relatively small dataset, several methods have been applied to derive an initial and final mass from fireball data. Two principal concepts have been used to estimate the mass of a meteoroid during its path through the atmosphere, the photometric and dynamic methods. Öpik [1] was one of the pioneers of the photometric method which uses the luminosity of the fireball to determine the incoming 'photometric' mass, and a corresponding luminous efficiency as a proxy for mass loss. Although analyses were conducted on the best data available, these datasets are limited in number and detail. It is therefore difficult to justify having "established a fully calibrated luminous efficiency parameter" [2]. The dynamic method uses the deceleration of the fireball to estimate mass. The accuracy to which deceleration could be measured from photographic plates was a limitation of this method [3, 4]. In recent years, Stulov [5] has combined unknown parameters in the dynamic equations into two dimensionless parameters to enable an easier analytical solution. This method was applied by Gritsevich [6, 7] to data sets which led to recovered meteorites. Although both methods have their limitations, a new approach to the dynamic method is presented here with the aim of completely automating the process of calculating the terminal bright flight mass. This calculation will form part of an automated workflow from fireball detection and triangulation through to orbit and fall calculation as part of the Australian Desert Fireball Network. The network will soon be expanding to a coverage area of 1.5-2 million km^2 with 50-60 camera stations. The data generated will exceed 100TB per year, and therefore automating the data pipeline – from event detection, image processing of the fireball track, triangulation, calculating terminal bright-flight mass, to darkflight and climate modelling – is a priority. The result will be automation of meteorite fall position estimates, with uncertainties, to facilitate rapid recovery of samples which may provide invaluable data for cosmo-chemists (particularly when combined with orbital data). This approach will also allow confirmation and refinement of mass and fragmentation models.

Model: The dynamic equations used for the entry of meteoroids through the atmosphere after [5] are

$$m\frac{dV}{dt} = -\frac{1}{2}c_d\rho_a V^2 S \quad (1)$$

$$H^*\frac{dm}{dt} = -\frac{1}{2}c_h\rho_a V^3 S, \quad (2)$$

where m is meteoroid mass, V is velocity, c_d and c_h the drag and heat coefficients respectively. H^* is the enthalpy of sublimation, ρ_a the atmospheric density and S the cross sectional area of the body which can also be written as

$$S = A\left(\frac{m}{\rho_m}\right)^{\mu}. \quad (3)$$

Here A is the shape parameter, ρ_m is the meteoroid density and μ the spin parameter.

Combining equation (3) with (1)-(2), the equations become

$$\frac{dV}{dt} = -\frac{1}{2}\frac{c_d\rho_a A}{\rho_m}V^2 m_m^{(\mu-1)} \quad (4)$$

$$\frac{dM}{dt} = -\frac{1}{2}\frac{c_h\rho_a}{H^*\rho_m}V^3 m_m^{\mu}. \quad (5)$$

The method proposed in this research takes a two-step approach. Initially a constrained dynamic optimisation is carried out to estimate the set of unknown parameters ($\frac{c_h}{H^*}$, A, ρ_m and μ) and the initial states (m_0, V_0). These parameters are then used to perform an extended Kalman filter estimation of the dynamic states (position, velocity and mass) from a sequence of position observations. Errors associated with the terminal mass and position are also calculated and are extremely valuable in estimating fall probabilities on the ground, which previous methods have not explicitly quantified.

Extended Kalman Filter. The objective of the Kalman filter is to estimate the states of a dynamic model based on a two-step process of "predict" and "update" [8]. In the meteoroid estimation problem, the states include the position along the line of trajectory, the velocity and mass of the meteoroid in the atmosphere. The prediction step involves estimating the state at an extrapolated time using the given dynamic model, including a covariance estimate for the estimated state vector. The update step refines the state estimate and covariance by taking an observation into account. An optimal "Kalman" gain is applied which takes into account both the uncertainty of the state estimate, and the uncertainty of the observation model.

Results: Ten data sets from the Australian Desert Fireball Network have been analysed in this way. The most complete, and therefore most reliable, being that of Bunburra Rockhole [9]. After the dynamic optimisation step, the optimal initial parameters are determined to be: $m_{0=}19.9\ kg$, V_0=13184 m/s, ρ_m=2000 kg/m^3, A=1.58, μ=2/3 and c_h/H^*=4.76x10^{-8} KJ/Kg. These were used in the subsequent extended Kalman filter to produce final states of m_{mf}= 0.909±0. 47 kg and V_f= 5792±309 m/s (with a larger assumed observation variance of 10000, top figure) and m_{mf}=1.08±0.47kg and V_f=5970±485 m/s (with a smaller assumed observation variance of 1000, bottom figure).

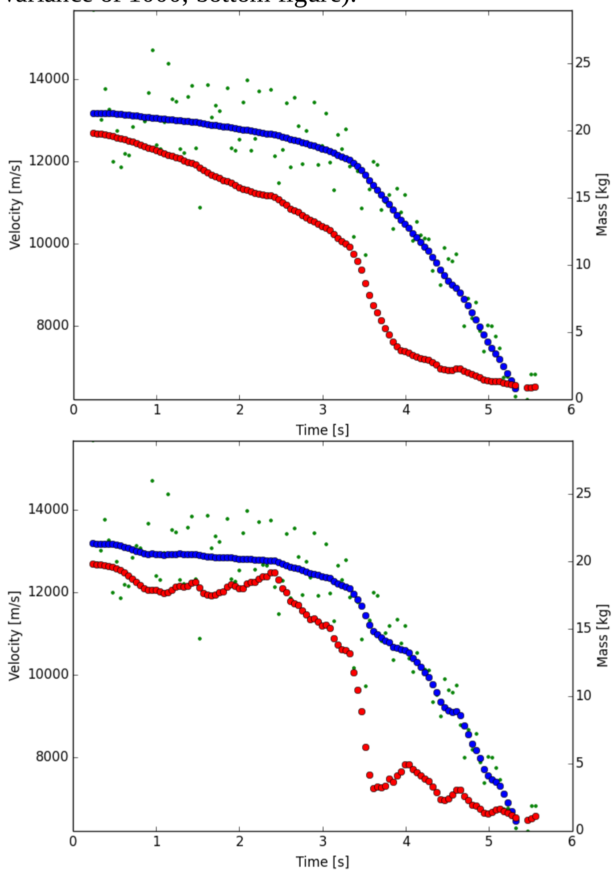

Figure: Results of Kalman filter step for Bunburra Rockhole. Blue shows velocity estimates as calculated by the extended Kalman filter over the raw velocity data (green). Red shows mass estimates along the trajectory. The two simulations apply different observation models.

Discussion: The Bunburra Rockhole data set, although very complete contains a very high initial scatter of ±3800 m/s in velocity. The Kalman filter is capable of producing optimal state estimates in the context of a noisy observation model.

The figure shows the velocities and masses estimated at each time step, considering the prior data. The locations where mass is greater than the previous timestep is a result of the values being updated by the present observation. The final state estimates and the corresponding covariance matrix will be used in subsequent dark flight calculations.

The Q_4 least-squares minimisation method produced by [10] was applied to this same data set. Smoothing was required before a result could be achieved due to the high scatter. All previous datasets analysed by this method have used fewer than 9 data points and are inherently smoothed. The results of this typical dynamic method using the same initial velocity and meteoroid spin as above gave α=27.434 and β=1.3221. When used in the following equation

$$m_f = exp\left(-\beta \frac{1-v_f^2}{1-\mu}\right) \quad \text{(eqn. 6 [10])}$$

a final mass of 2.23 kg is achieved. An initial mass of 19.32 kg was also calculated (eqn. 12 [10]). Both this method and the proposed method give similar results to initial and final bright-flight masses published by [9] (22.0±1.3 kg and 1.1 kg respectively). The Q_4 least-squares minimisation method however requires ad-hoc prior smoothing of noisy data and does not produce explicit error estimates.

Conclusion: The two-step approach proposed here provides a rigorous method for determining the terminal bright-flight mass for use in the automation of the Australian Desert Fireball Network. The extended Kalman filter is applied to raw input data with high variance to estimate the state of a fireball at each time step and give optimal estimate of the terminal bright-flight position, mass and velocity with error estimates. This coupled with the dynamic optimisation for determining unknown input parameters makes it a good method of analysing fireball bright flight trajectories. Currently the approach is limited by the absence of a fragmentation model, and there is also work to be done in refining the parameterization of the dynamic model, but our preliminary results are encouraging. The goal is that future iterations of the model will constrain fragmentation.

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