FREE LIBRATION MODES OF A STRUCTURED MOON. James G. Williams, Dale H. Boggs, and J. Todd Ratcliff, Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA, 91109, USA (e-mail James.G.Williams@jpl.nasa.gov).

Introduction: Lunar physical librations, the variations in rotation and in orientation of the pole, are sensitive to lunar structure. The forced physical librations arise from time-varying torques about the three axes. The free libration modes are normal modes and their periods are resonances for the forced librations. The three mantle modes are familiar. There is a libration in longitude, a wobble of the body's pole direction with respect to its rotation axis, and a free precession of the pole in space. A fluid core introduces another free precession mode often called the free core nutation. But what do we expect for the inner core? We know little about the inner core, so any information would be welcome. Unlike the Earth, the Moon's figure is not near an equilibrium shape for its present orbit and spin rate.

The amplitude and phase of each free mode must be determined from observations. They depend on the initial conditions at some epoch. Three Euler angles and three spin rate components specify the initial conditions for the 3-axis orientation of crust plus mantle. An analogous set of three angles and three spin rate components would specify the initial conditions for the inner core. The fluid interactions depend on the three components of the spin rate vector of the fluid, but not on its Euler angles. 15 initial conditions are required for crust+mantle, fluid outer core, and solid inner core. Since two initial conditions are required to determine the amplitude and phase of each mode, then 7.5 modes are expected!

A recent analysis of the forced and free libration amplitudes, phases, and periods was made by Rambaux and Williams [1]. That study extracted a series of periodic terms by analyzing the numerically integrated DE421 physical librations [2] that resulted from a fit to Lunar Laser Ranging (LLR) observations.

Mantle Modes: The three modes for mantle+crust are familiar. (1) The libration in longitude is a sinusoidal speeding up and slowing down of the rotation about its polar axis, the axis corresponding to the largest moment of inertia. LLR data analysis finds a period of 1056.1 d and an amplitude of 1.30″, or 11 m at the equator [1]. (2) The wobble mode is analogous to the Chandler wobble of the Earth. Viewed from the rotating Moon, the mean axis of rotation appears to trace out an elliptical path about the polar axis, but equivalently, the pole can be considered to wobble about the axis of rotation. LLR finds semiaxes of 3.3″x8.2″ (28 m x 69 m) and a period of 74.6 yr [1]. (3) The latitude mode is a retrograde free precession of the pole in space. The period is 80.9 yr. Although [1] saw a small term when analyzing DE421 physical librations, the amplitude was small and it was considered uncertain.

Outer Core Modes: The fluid rotation can be different from the solid rotation. (4) One free mode is a retrograde precession of the spin rate vector of the outer core with respect to space. The analogous mode for the Earth is often called the free core nutation. The free period depends on the oblate shape, the flattening, of the outer core boundaries. The amplitude and phase of this mode are not detected by the LLR analyses. However, the flattening of the core-mantle boundary (CMB) is detected, but that value depends on the core moment, which is imperfectly known. The precession period is expected to be >1 century, and it is difficult to observe. (5) The third initial condition corresponds to a fluid rotation rate that does not match the synchronous rate. There is no phase, hence the half mode.

Core Oblateness: Torque from an oblate CMB shape is proportional to the difference between the fluid outer core's polar and two equatorial moments, $C_f-(A_f+B_f)/2$, provided that the fluid has uniform density and the inner boundary is spherical. This moment difference depends on the product $f\,C_f$ of the fluid core moment of inertia C_f and the CMB flattening f, where $f = [C_f-(A_f+B_f)/2]/C_f$ and $f\,C_f/C = [C_f-(A_f+B_f)/2]/C$. The recently determined DE430 flattening $f = (2.46\pm1.4)\times10^{-4}$ [3, 4] is based on an LLR solution. The fluid precession frequency, proportional to f, gives a 300-yr fluid precession period, but smaller cores give shorter periods.

Inner Core Modes: The three inner core modes are analogous to the mantle modes: (6) a longitude libration, (7) a wobble mode, and (8) a free precession. None of these three have been observed.

Damping: Peale [5] made an early study of damping of free modes. Damping expressions for the three mantle modes and the core free rotation mode were derived by Williams et al. [6]. The fluid rotation damps with a ~120 yr time scale, so it is not expected. The longitude libration damps in ~25,000 yr and the wobble damping time is roughly a million years, but lack of knowledge of the tidal Q at 74.6 yr makes the latter very uncertain. The damping of the precession mode is 1.4×10^5 yr (the damping equations in [6] are correct, but there is an incorrect exponent for the numerical value given at the end of section 20).

Theory and Model Calculations: The Moon has a small core with a moment of inertia $<10^{-3}$ of the total moment. Consequently, the mantle modes are only slightly perturbed by the outer and inner core, and the modes for a completely solid Moon are a first approximation; see Eckhardt for solid Moon theory [7]. The

inner core modes are strongly affected by the attraction between the mantle and inner core. Torques between the inner core and mantle are expected to be stronger than those between the inner core and the Earth. Although torques between the inner core and mantle have a minor effect on the mantle modes, they have a major effect on the inner core modes. The coupling comes from the internal gravity field below the CMB and the exterior gravity field from the inner core. Apart from the LLR result for flattening of the CMB, we do not have the information that we need. Without further observational information, we must make model calculations.

There is information on the external gravity field from GRAIL data analysis [8, 9] and on the surface topography from LOLA [10, 11]. For model calculations we follow Meyer and Wisdom [12] by assuming that the external gravity field arises near the surface. We can include or exclude surface topography. The near surface gravity field is extended down to the core. The shapes of the two core boundaries deform so that the potentials on their surfaces are constant. Then the gravitational interactions between inner core and mantle and inner core and Earth can be calculated. The model normal mode calculations include periods and eigenvectors.

Longitude Libration Modes: Model calculations indicate that the 2.9 yr mantle mode involves mantle and inner core liberating in phase, but with different amplitudes. The inner core mode has a model period of roughly 2 yr with most of the motion in the core. Van Hoolst et al. [13] have developed a theory for the librations of an icy shell and a solid core, an analogous situation.

Precession Modes: The mantle free mode has the three units precessing together with similar mantle and inner core amplitudes, but the fluid core tilt has reversed sign. The outer and inner core modes each involve precession of those units with only minor involvement of other layers. The outer core mode has a period of 80 yr to several centuries whereas the inner core mode is roughly a decade. We do not confirm the long inner-core precession periods found by Gusev and Petrova [14]. They apply a theory developed for the Earth. Their fluid precession periods are compatible, but we find a greater spread of periods than they indicate.

Wobble Modes: The wobble of the crust plus mantle is the familiar elliptical motion of the pole with a 74.6 yr period. The wobble of the mantle and inner core are coupled. The inner core wobble has a model period of about a decade. The inner core period is shorter than the values given by Gusev and Petrova [14].

Summary: LLR analysis may be able to detect the influence of the inner core on the physical librations, but that influence is subtle if detectable. Even if the inner core free librations have amplitudes too small to detect, their periods are resonances for forced librations. By predicting the resonance periods, the model calculations help localize the periods of forced libration terms that are most useful to search.

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