CHARACTERIZATION OF LUNAR CRATER WALL SLUMPING FROM CHEBYSHEV APPROXIMATION OF LUNAR CRATER SHAPES P. Mahanti[1], M.S. Robinson[1], T.J. Thompson[1] [1]Lunar Reconnaissance Orbiter Camera, School of Earth and Space Exploration, Arizona State University, Tempe,AZ,USA; pmahanti@asu.edu;

Introduction: Impact events continue to reshape the lunar landscape; shapes of impact craters evolve over time as a result of both slow (e.g. micrometeorite impacts) and instantaneous (e.g. impact events occurring on existing craters) processes. The shapes of craters are thus key indicators of the nature of past and present surface processes as well as the target property itself. Visibly, from high resolution images, lunar craters exhibit a wide variety of forms - overall shape, size and topography of one crater being significantly different from another, making the process of comparing crater shapes complex. While crater depths and diameters are often used to compare craters, these simple measurements do not describe the detailed shape of a crater necessary for correlating crater shapes to surface processes over time. One example of such a process is the slumping of crater rim/wall where dislodged material (blocks and fine-grained) from crater rim and wall collects under the forces of gravity and friction. The slump formed can be identified and characterized from the topographic profile of the crater, only the measurements of the observed depth and diameter of the crater is not sufficient. Further, if the slumping is locally constrained (e.g. present in the north-east but not elsewhere) then this results in an asymmetric topographic profile (when drawn north-east to south-west) which can be compared with other parts of the crater to analyze the slumping.

The topography of nearly all lunar craters can be represented by a relatively small set of the Chebyshev polynomials [1]. In this representation, the individual polynomial functions are scaled and summed to approximate the actual crater elevation profile (Figure 2B shows an order 16 approximation of the Tycho crater), and the scaling factors are the Chebyshev coefficients themselves. In this work we show that the symmetry properties of Chebyshev polynomials can be used to characterize effects of mass wasting processes on crater walls and Chebyshev coefficient values can be used to detect and characterize processes such as crater wall slumping from digital terrain models.

Chebyshev Polynomials and Symmetry properties: Chebyshev polynomials are a series of orthogonal polynomials that are commonly used to retrieve least-square-error function approximations. Orthogonal polynomial series members have a unique shape (not correlated with any other in the series) and satisfy a property that their inner-product (similar to dot product for a vector) is zero. The unique shapes of the member polynomials can be used as fundamental building blocks for

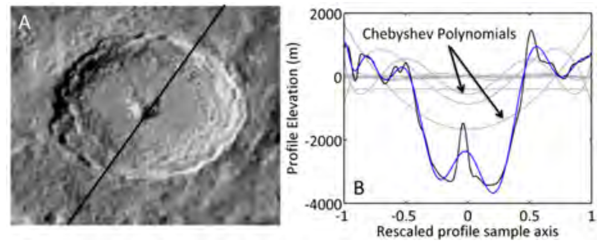

Figure 1: (A) Tycho crater with elevation profile. (B) Order 16 approximation of Tycho crater profile (shown in blue), the corresponding Chebyshev polynomials are shown in the background.

a complex function shape. Chebyshev polynomials can be defined recursively where the expression of one polynomial leads to the next higher order polynomial. The polynomial for degree 'n' is denoted $T_n(x)$ (x is the sample or reference axis), and is given by the recurrence relation:

$$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x); |x| \leq 1 \quad (1)$$

with $T_0(x) = 1$ and $T_1(x) = x$ being the first two member polynomials (higher order polynomials shown in Table 1 [1]). The individual Chebyshev polynomials each represent a component of the overall crater shape (such as, part of the crater rim, wall, or floor) and, combined with the corresponding coefficients, describes a particular crater. Unique and physically meaningful analysis of crater topography can be achieved by the use of Chebyshev polynomials.

Chebyshev polynomial series members are alternately even (symmetric) and odd (anti-symmetric) polynomial functions. If the value of n is even, then the corresponding Chebyshev polynomial T_n is also even and a symmetric function (symmetric about the vertical axis). Similarly, T_n is an odd (anti-symmetric) function if n is odd. Only the odd Chebyshev polynomials pass through the origin (they have $x = 0$ as a root). Due to the above property of the Chebyshev polynomials, the odd Chebyshev coefficients are associated with asymmetry that is observable in the topographic data.

Methods, Results and Discussions: In this work we are specifically interested in crater walls with asymmetric topography - craters where one part of the crater wall is asymmetric while the other parts of the crater are radially symmetric. The disparity in symmetry can be shown by obtaining 4 diameter profiles (Figure 2): southwest to northeast (z1), south to north (z2), west to east (z3), and southeast to northwest (z4). If the

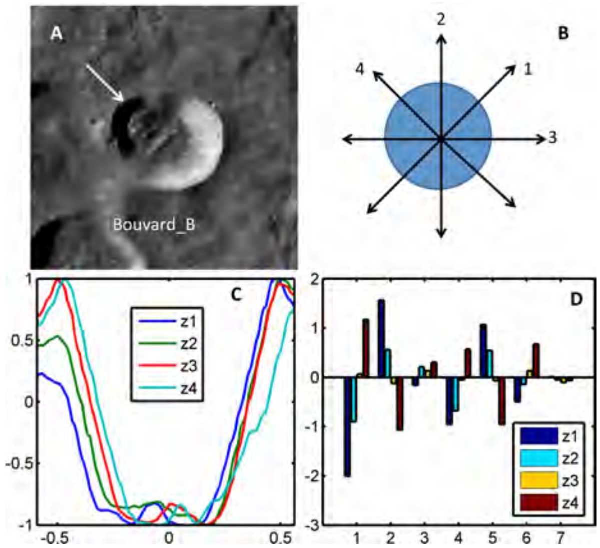

Figure 2: (A)Bouvard B crater showing the location of wall sluming. (B) The four directions for elevation profiles (C) Elevation Profiles and (D) Odd Chebyshev coefficients for Bouvard B

asymmetry is very localized, then only one profile is affected, otherwise three profiles are affected. For example, for crater Bouvard B (−41.7°N, 280.15°E), only the 4th elevation profile is affected due to the asymmetry, however, if not localized, profiles 2 and 3 can also be affected (have asymmetric shapes of varying degrees). The four profiles are then analyzed using the Chebyshev approximation method and the odd Chebyshev coefficients are compared to analyze the affect of asymmetry.

The crater elevation profiles are obtained from the WACGLD100 digital elevation model [2] which is based on the images acquired by Lunar Reconnaissance Orbiter Camera Wide Angle Camera (LROC WAC) [3]. For slumping and other mass wasting processes that affect the topography of smaller craters (diameter 1 km and below), high resolution topography (e.g. LROC NAC DTMs, 2 to 5 m pixel scales) from LROC NAC DTMS can be used without changing the method of analysis.

For the Bouvard B crater, z4 (Figure 2C) is the profile that has the strongest indication of slumping, which appears as a break in the crater wall. When the odd Chebyshev coefficients corresponding to Chebyshev approximation of z1,z2, z3, z4 are compared (Figure 2D), the coefficient corresponding to z4 is found to be different. Only the first 7 odd Chebyshev coefficients are shown here i.e. coefficient numbers 1,3,5,7,9,11, and 13 of the approximation.

For the Drude crater (−38.62°N, 268.12°E), except z1 (Figure 3A) all profiles show indications of slumping. Accordingly the first 4 odd coefficients (coefficient numbers 1,3,5,7) distinctly show the difference between z1 and the group z2,z3,z4 (Figure 3B). Note that lower numbered coefficients have higher contribution to the overall topographic shape.

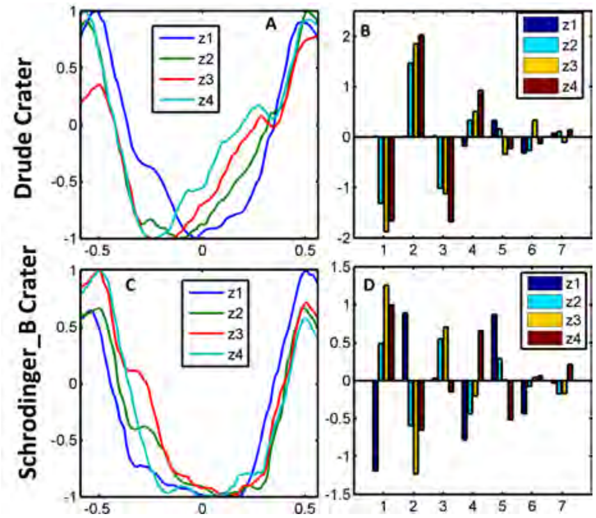

Figure 3: Four profiles and odd Chebyshev coefficients for Schrodinger B and Drude craters

For the Schrodinger B crater (−68.06°N, 141.44°E), except z4 (Figure 3C) all profiles show some indications of slumping. As in the case of Drude, the first 4 odd coefficients (coefficient numbers 1,3,5,7) distinctly show the difference between the group z1,z2,z3 and z4. Note that for Drude and Schrodinger B, the slumping effect is present at different portions of the craters.

Conclusion: Mass wasting processes were active as the topography of the Moon evolved and there is evidence of processes like landslides in recent times [4]. Local asymmetry in crater wall topography can be a result of a process like slumping after formation or may be caused due to impact from smaller craters. In either case, the localized asymmetry represents a change in the impact crater topography after formation and can be detected and characterized via the use of Chebyshev polynomials.

References: [1] P. Mahanti, et al. (2014) *Icarus* 241:114. [2] F. Scholten, et al. (2012) *Journal of Geophysical Research: Planets (1991–2012)* 117(E12). [3] M. Robinson, et al. (2010) *Space science reviews* 150(1):81. [4] P. Senthil Kumar, et al. (2013) *Journal of Geophysical Research: Planets* 118(2):206.