DEVELOPMENT OF A MODEL PREDICTIVE CONTROL BASED ALGORITHM TO A TARGET-CHASER RENDEZVOUS MANEUVERING SCENARIO

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ABSTRACT

The utilization of space for human activities has been expanding steadily over the past several decades. This in turn has also increased the amount of space debris, inducing potential risks for space operations. Hence, space debris removal has become an important topic of discussion. They can be considered uncooperative objects orbiting the earth with arbitrary motions. Debris removal satellites need to follow this motion carefully to deploy grappling mechanisms. This study addresses this problem with a model predictive control-based algorithm to efficiently track the object, while considering fuel consumption and thruster limitations.

1 INTRODUCTION

In recent years, advancements in space development have increased with collaborative research activities at the International Space Station (ISS), satellite-based network communication, ongoing Mars and Moon-based missions as well as enrollment of private organizations in space-related activities. However, this is hampered by the increase in space debris throughout the years, such as debris in the Earth's orbit consisting of defunct satellites, rockets upper stages, and fragments left from collisions. These could fly at speeds as much as 7 km/s; hence, even a small fraction of a part could adversely affect functioning satellites and other space crafts. As of 2019, the count of large-sized space debris exceeded 19,000 [1]. With the probability of collisions expected to rise in the future, prevention of the formation of future debris is insufficient. Active space debris mitigation is necessary to contain an increase in the total number of debris objects.

Currently, attempts made by public and private institutions in countries worldwide are underway to tackle the space debris problem. The Japan Aerospace Exploration Agency (JAXA) and the European Space Agency (ESA) are among the public organizations that have initiated debris removal methods. JAXA proposed attaching a tether to debris as a possible solution in the KITE mission [2] conducted in 2016. ClearSpace-1 [3] is supervised by ESA to conduct space debris removal using a satellite equipped with robotic arms. Private companies, such as Kawasaki Heavy Industries and Astroscale, are also conducting demonstrations to find viable solutions to the space debris problem. The former has a planned demonstration mission, Debris Removal Unprecedented Micro-Satellite (DRUMS) to capture a mock object using an extendable boom as the capture mechanism [4]. ELSA-d introduced by Astroscale is based on magnetic structures placed on the target to retrieve through the capturing process [5].

In the above situations, a thrust is generated by the propulsion system of the satellite to deorbit the debris once it is captured by a grappling mechanism. Debris is an uncooperative object with different rotational motions. Chaser satellites should fly in such a way that both the target body and the chaser align in a specific way for successful capture. Therefore, the relative rotational motion between the contact point of the debris and the capturing mechanism of the chaser are canceled out. A number of studies have been conducted regarding the relative motion between objects in space. Reference [6] described using the state-dependent Riccatti equation-based control and linear quadratic regulator (LQR)-based control to maintain a constant distance from a target body, while minimizing the fuel consumption and settling time. Reference [7] investigated the continuous trust Clohessy–Wiltshire (C–W) model with varying initial conditions for low-thrust relative motion. Here, the optimization is performed considering state and co-state vectors and formulated as a two-point value problem.

With the limited amount of resources in space and restrictions caused by propulsion systems, these optimization strategies must be constrained when designing a control system [8]. In such cases, model predictive control (MPC) is an attractive alternative with its constraint optimization capabilities. An MPC-based solution was derived for a rendezvous and docking scenario with a tumbling target in [9] with several quadratic and time-varying constraints in the optimization problem. In [10], a similar case study without tumbling effects is considered with a cost function based on the unconstrained linear quadratic problem for guaranteed stability.

In this study, an MPC controller based on constrained optimization is used to perform the orbital rendezvous for a target chaser situation. The methodology explains the relative motion in space and controller designs. The results and discussion present the computer simulations and comparisons between several controllers. The overall performance is summarized in the conclusions.

2 METHODOLOGY

2.1 Equations of Relative Motion

Fig. 1 denotes the moving reference frame relative to the target. The orbit radial direction is set as the x-axis and z-axis points to the angular momentum vector, which is orthogonal to the orbital frame. The y-axis completes the coordinate system by forming a right-hand orthogonal system. r_t and r are the position vectors of the target and chaser, respectively, regarding the center of the Earth. ω is the orbital angular velocity of the target.

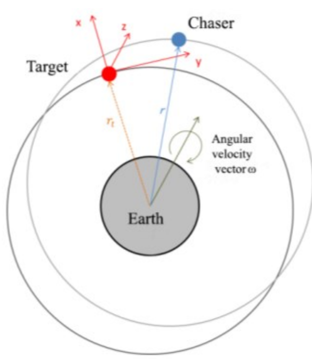

Figure 1. Local Vertical Local Horizontal Frame with Origin at the Target

If the perturbations are neglected, the relative motion dynamics can be considered by solving the following second-order differential equations: (x, y, z) are the position coordinates, and ($\dot{x}$, $\dot{y}$, $\dot{z}$) correspond to the relative velocities. μ is the geocentric gravitational constant, α_x, α_y and α_z are the acceleration components of the chaser.

$$\ddot{x}\text{-}2\omega\dot{y}\text{-}\dot{\omega}y\text{-}\omega^2x=-\frac{\mu}{r^3}(x+r_t)+\frac{\mu}{r_t^2}+\alpha_x \quad (1)$$

$$\ddot{y}+2\omega\dot{x}+\dot{\omega}x\text{-}\omega^2y=-\frac{\mu}{r^3}y+\alpha_y \quad (2)$$

$$\ddot{z}=-\frac{\mu}{r^3}z+\alpha_z \quad (3)$$

When the distance between the chaser and target is relatively small compared to the distance between the target and the center of the Earth and the target orbit is of circular motion, C–W or Hill's equation can be derived as [11],

$$\ddot{x}=3\omega^2x+2\omega\dot{y}+\alpha_x \quad (4)$$

$$\ddot{y}=-2\omega\dot{x}+\alpha_y \quad (5)$$

$$\ddot{z}=-\omega^2z+\alpha_z \quad (6)$$

Using the state variables (x, y, and z; $\dot{x}$, $\dot{y}$, and $\dot{z}$), the continuous-time state equation can be introduced in the form of

$$\dot{x}=Ax+Bu \quad (7)$$

Where,

$$x=\begin{Bmatrix} x \\ y \\ z \\ \dot{x} \\ \dot{y} \\ \dot{z} \end{Bmatrix}$$

$$A=\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 3\omega^2 & 0 & 0 & 0 & 2\omega & 0 \\ 0 & 0 & 0 & -2\omega & 0 & 0 \\ 0 & 0 & -\omega^2 & 0 & 0 & 0 \end{bmatrix} \quad (8)$$

$$B=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$u=\begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{Bmatrix}$$

2.2 Model Predictive Control (MPC)

MPC is a control method in which the future behavior of the manipulated control quantity is predicted to optimize the future behavior of the system output within a limited time window. This is repeated for each time step to revise the manipulation quantity. Generally, the method used to determine the manipulation quantity is based on the optimization problem. When solving optimization problems that minimize the evaluation function for a certain interval, the control quantity is expressed in terms of the manipulation quantity using the state equation (dynamic model). The advantage of MPC over other control methods is that, although the state and input constraints are limited to closed set constraints, the target can be controlled while explicitly stating the input.

The discrete time states pace model is described below.

$$x_p(k+1)=A_p x_p(k)+B_p u(k) \quad (9)$$

$$y(k)=C_p x_p(k) \quad (10)$$

where $x_p(k)$ and u(k) are the state quantity vector and control input vector at time step k, respectively. A, B, and C are the state, input, and output matrices, respectively. Moreover, by defining the difference in the state variable delta and the control input variable delta as

$$\Delta x_p(k)=x_p(k)-x_p(k-1)$$

$$\Delta u(k)=u(k)-u(k-1)$$

the increment in the state and output can be written as

$$\Delta x_p(k+1)=A_p \Delta x_p(k)+B_p \Delta u(k) \quad (11)$$

$$y(k+1)-y(k)=C_p A_p \Delta x(k)+C_p B_p \Delta u(k) \quad (12)$$

Considering a new state variable using

$$x(k)=\begin{bmatrix} \Delta x_p(k)^T \\ y(k) \end{bmatrix}^T$$

Eq. 9 and Eq. 10 can be rewritten as

$$x(k+1)=Ax(k)+B\Delta u(k) \quad (13)$$

$$y(k)=Cx(k) \quad (14)$$

with the new matrices being,

$$\mathbf{A}=\begin{bmatrix} A_p & 0_p{}^T \\ C_p A_p & 1 \end{bmatrix}$$

$$\mathbf{B}=\begin{bmatrix} B_p \\ C_p B_p \end{bmatrix} \quad (15)$$

$$\mathbf{C}=[0_p \quad 1]$$

The following optimization problem to minimize the evaluation function can then be solved for each time step. Here $r \in R^n$, $Q \in R^{n\times n}$ and $R \in R^{n\times n}$ are the reference input and weight coefficients, respectively. Hp and Hu are the prediction horizon and control horizon, respectively.

$$\min_{\Delta u} \sum_{i=1}^{Hp} [x(i|k)-r(i|k)]^T Q [x(i|k)-r(i|k)] + \sum_{i=0}^{Hu-1} \Delta u(i|k)^T R \Delta u(i|k) \quad (16)$$

such that (s.t.)

$$x(k+1)=\boldsymbol{A}x(k)+\boldsymbol{B}\Delta u(k) \quad (17)$$

$$u_{max} \geq u \geq u_{min} \quad (18)$$

With the control input column Δu denoted by

$$\Delta U(k)=[\Delta u(k),.....,\Delta u(k+Hu-1)]^T \quad (19)$$

the optimization problem is given by,

$$\min_{\Delta U(k)} V(=\frac{1}{2}\Delta U^T H \Delta U+\theta^T(k) F \Delta U) \quad (20)$$

s.t.

$$G\Delta U \leq W + S\theta(k) \quad (21)$$

With the variables being, Q=diag[] $\in R^{Hp\times Hp}$, R = diag[] $\in R^{Hu\times Hu}$, $G \in R^{2mHu\times mHu}$, $W \in R^{2mHu\times 1}$, and $S \in R^{2mHu\times(2n+m)}$.

$$F=\begin{bmatrix} 2\varphi^T Q \emptyset \\ 0_m \\ -2Q\emptyset \end{bmatrix}, \theta(k)=\begin{bmatrix} x(k) \\ u(k-1) \\ r \end{bmatrix}$$

$$H=2R+2\emptyset^T Q \emptyset$$

$$\varphi=\begin{bmatrix} CA \\ \vdots \\ CA^{Hp} \end{bmatrix} \quad (22)$$

$$\emptyset=\begin{bmatrix} CB & 0 & \cdots & 0 \\ CAB & CB & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ CA^{Hu-1}B & CA^{Hu-2}B & \cdots & CB \end{bmatrix}$$

With reference to Eq.

(20), the quadratic problem can be considered to be one of minimizing V by updating the initial value θ(k) at every time step, while assuming ΔU(k) as a variable. Here, if the Hessian matrix H is symmetric and semi-positive definite, V can be regarded as a convex function. In addition, if the constraint ($G\Delta U \leq W + S\theta(k)$) is a closed set, a global optimal solution can be obtained for this optimization problem using quadratic programming-based solvers.

2.3 Linear Quadratic Regulator (LQR)

Linear Quadratic Regulator or LQR is a type of optimal controller. It is a type of a state feedback controller which measures the states of the system and then generates a response leading the states to zero while minimizing the cost function J given below.

$$V=\sum_{n=0}^{\infty} x^T Q x+u^T R u \quad (23)$$

Q and R denote the weighting matrices corresponding to the states and input matrices, respectively. The optimal gain K is calculated by solving the discrete time algebraic Riccati equation.

$$K=(B^TXB+R)^{-1}B^TXA \tag{24}$$

$$A^TXA-X-A^TXB(B^TXBR)^{-1}\ B^TXA+Q=0 \tag{25}$$

2.4 C–W Based Control

Assuming a relative position $r(x_0,y_0,z_0)$ and a relative velocity $\dot{r}(\dot{x}_0,\dot{y}_0,\dot{z}_0)$ at a given time $t=t_0$, the change in velocity, ΔV, required to perform a rendezvous maneuvering to an arbitrary position after $t=t_0+\tau$ can be calculated based on the C–W solution as follows [12].

$$\begin{Bmatrix}\Delta V_x\\ \Delta V_y\\ \Delta V_z\end{Bmatrix}=\frac{\omega}{K}A\begin{Bmatrix}x_0\\ y_0\\ z_0\end{Bmatrix}-\begin{Bmatrix}\dot{x}_0\\ \dot{y}_0\\ \dot{z}_0\end{Bmatrix} \tag{26}$$

where,

$$A=\begin{bmatrix}A_1 & A_2 & 0\\ A_3 & A_4 & 0\\ 0 & 0 & A_5\end{bmatrix}$$

$$A_1=4\sin\omega\tau-\omega\tau\cos\omega\tau$$

$$A_2=-2(1-\cos\omega\tau)$$

$$A_3=-6\omega\tau\sin\omega\tau+14(1-\cos\omega\tau)$$

$$A_4=\sin\omega\tau$$

$$A_5=-\frac{K}{\tan\omega\tau}$$

2.6 Quantizer Design

As the outputs from the three controllers are continuous values, a static quantizer is used to simulate the ON/OFF input constraint, as follows.

$$u=\begin{cases}u_{max} & (u\geq u_{max}/2)\\ 0 & (u_{max}/2>u>u_{min}/2)\\ u_{min} & (u\leq u_{min}/2)\end{cases} \tag{27}$$

The flow diagram representing the total control flow is demonstrated in Fig. 2. The reference input is fed into the system, and the controller generates the desired output. This is quantized and sent to the satellite model for the simulation. The output from the model is then sent back to the system as feedback to calculate the error.

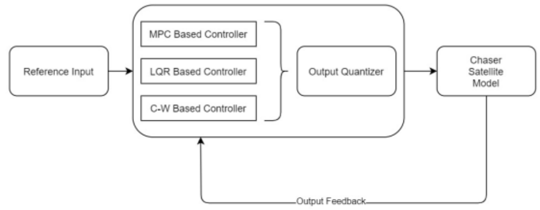

Figure 2. Control Flow Diagram

3 RESULTS AND DISCUSSION

The simulation conditions for the three control strategies are given in Tab. 1. For the flyaround scenario, only the XY plane is considered in this situation.

Table 1: Simulation Conditions

Parameter	Value
Orbit period	400 s
Orbit radius	4m
Initial values $(x_0, y_0, \dot{x}_0, \dot{y}_0)$ [m, ms^{-1}]	(0,-5,0,0)
Input constraints [mms^{-2}]	$\lvert u_x\rvert \leq 2.88$ $\lvert u_y\rvert \leq 2.88$
Coefficient matrix Q	diag [1 1 1 1]
Coefficient matrix R	diag [1 1]
Prediction horizon	15
Control horizon	3
Discrete time period	1(CW),2(MPC)

The simulation time was fixed for 1000 s, and each controller response is as shown in Fig. 3. Fig. 3 corresponds to the change in the flyaround orbit of the three scenarios considered.

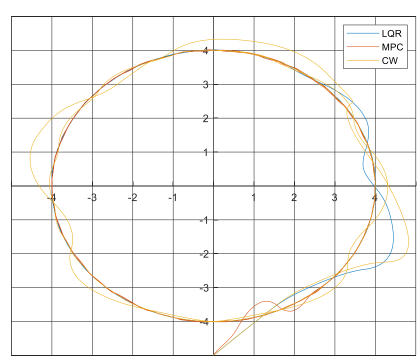

Figure 3: Comparison of the Fly around Orbits with Three Controllers

Figs. 4–6 show the change in radius with respect to time with the C–W, LQR, and MPC controllers.

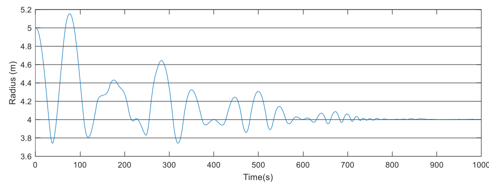

Figure 4. C–W- Change in Radius with Time

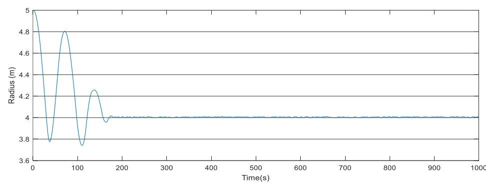

Figure 5. LQR- Change in Radius with Time

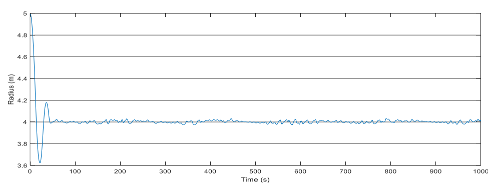

Figure 6. MPC- Change in Radius with Time

The angular velocity changes over the simulated time among the three controllers are denoted in Figs. 7–9.

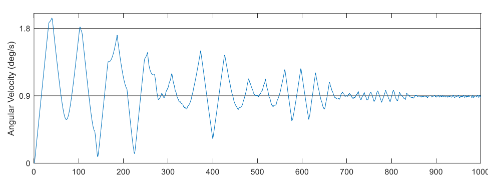

Figure 7. C–W- Change in Angular Velocity with Time

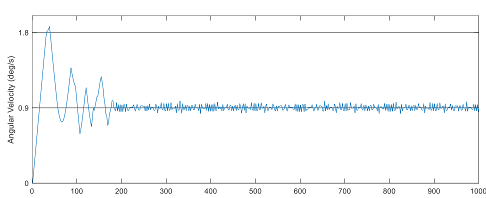

Figure 8. LQR- Change in Angular Velocity with Time

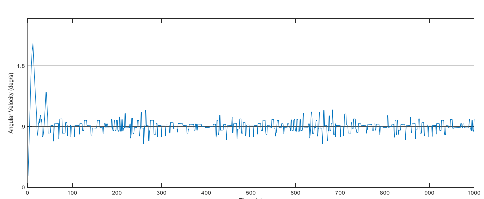

Figure 9. MPC- Change in Angular Velocity with Time

The commanded control signal and quantized control input for the X and Y axes for each controller are presented in Figs. 10–15.

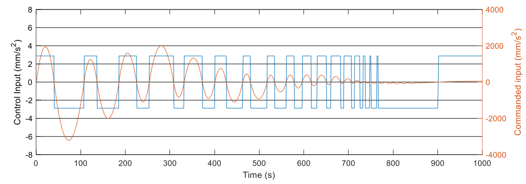

Figure 10. C–W-X Axis: Command Inputs with Time

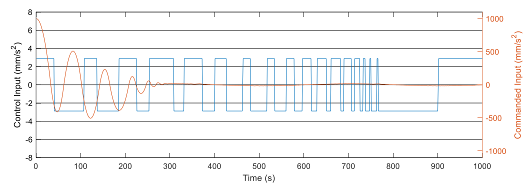

Figure 11. C–W-Y Axis: Command Inputs with Time

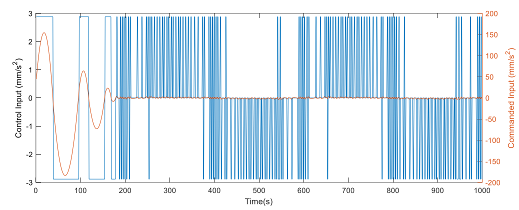

Figure 12. LQR-X Axis: Command Inputs with Time

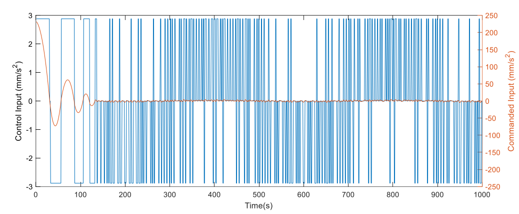

Figure 13. LQR-Y Axis: Command Inputs with Time

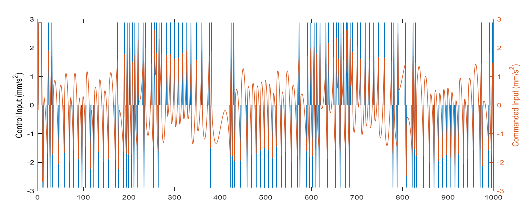

Figure 14. MPC-X Axis: Command Inputs with Time

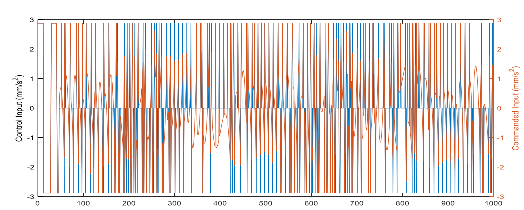

Figure 15. MPC-Y Axis: Command Inputs with Time

The root mean square errors for the position and angular velocity corresponding to each controller are listed in Tab. 2 The total ΔU requirements for each controller for both axes are given in Tab. 3.

Table 2: Root Mean Square Errors (RMSE)

Controller	RMSE Values			
	x	y	$\dot{x}$	$\dot{y}$
C–W	0.3143	0.1909	0.0221	0.0135
LQR	0.1739	0.1428	0.0122	0.0089
MPC	0.0738	0.1215	0.0225	0.0228

Table 3: Total ΔU for Each Controller after 1000 s.

Controller	ΔU Required (mm/s)	
	X axis	Y axis
C–W	2.877	2.765
LQR	1.549	1.731
MPC	0.426	0.878

4 CONCLUSIONS

This study aimed to develop a control strategy for the flyaround orbit control scenario between a chaser and a target pair of bodies. The equations pertaining to relative motion in space were derived, and an MPC-based algorithm was developed. The algorithm was aimed at minimizing the fuel consumption, while realizing the required orbit control. From Tab. 2 and Tab. 3, it can be concluded that the MPC-based controller has an advantage over conventional controllers in delivering faster convergence with minimum fuel usage. The MPC results can be further improved by increasing the prediction and control horizons. However, this requires higher processing power of the onboard computer system, which is a significant drawback and needs to be addressed with a different approach.

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