ON THE PREDICTION OF KEYHOLES BY PROPAGATION OF THE MOID.

O. Fuentes-Muñoz[1], A. Pedrós-Faura[1], D. Amato[2], D. J. Scheeres[1], J. W. McMahon[1]
[1]Colorado Center for Astrodynamics Research, University of Colorado Boulder
[2]Department of Aeronautics, Imperial College London

Keyholes are regions in the vicinity of Earth such that if an asteroid crosses them during a close encounter, the asteroid will collide with Earth in a future encounter. When an asteroid is discovered during a close encounter, its orbit is determined with large uncertainty until further observations are available. Then, the probability of collision is estimated and made available publicly[1]–[3]. This is the case of many asteroids, including (99942) Apophis. The orbit solution of (99942) Apophis was initially compatible with a few collision trajectories. In other words, it was possible that (99942) Apophis had crossed an Earth keyhole.

The analytical Öpik theory [5]–[7] computes the outcome of a planetary close encounter as a function of the coordinates in the planetocentric target plane or B-plane, shown in Figure 1. One of the results is the definition of the Valsecchi circles, the loci of the conditions for a resonant encounter[7], which are also shown in Figure 2. A resonant encounter implies that the subsequent encounter occurs after an integer amount of years of both the asteroid and the planet. If, in addition, the Minimum Orbit Intersection Distance (MOID) at the resonant encounter is smaller than the collision radius, the asteroid is crossing a keyhole. The MOID is typically considered constant or linearly drifting between close encounters. However, the variation in Earth MOID in a few years can be of the order of magnitude of many Earth radii. This means that in order to find keyholes on the B-plane we need to find the variation in the MOID after the asteroid crosses a given point in the B-plane.

In this study, we compare the different methods to propagate the orbits of asteroids between planetary close encounters. This indicates how well these tools can estimate if an asteroid crossed an Earth keyhole during a close encounter. Among the possible tools, we compare analytical solutions, semi-analytical, and fully numerical integrations. Analytical methods allow us to estimate the long-term dynamics of NEOs in a fast way but usually rely on certain limiting assumptions. Numerical integration is more computationally intensive and if very high fidelity is required, detailed knowledge of the asteroid's physical characteristics is also required[4]. When this knowledge is not available, a generic approach that benefits from the fast analytical propagation techniques can be particularly useful.

In this work we study the variation of the MOID between encounters so that we can determine the actual keyholes that exist in the B-plane. Depending on the heliocentric orbit of the asteroid we can use analytical secular theories that track the secular evolution of the MOID beyond a constant linear drift. Here we implement a solution of the Laplace-Lagrange theory that accounts for the secular drift due to the gravitational force of Jupiter[8].

Superimposed on the secular drift in the MOID, there are short-period oscillations that can be of the order of magnitude of several Earth radii. This means that there are regions in which the exact variation of the MOID can only be estimated accurately by numerical integration.

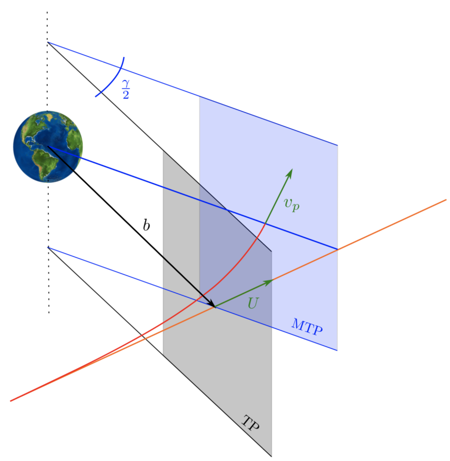

Figure 1. Planetary flyby target plane (TP), defined by the relative velocities U at the asymptotic velocity and Modified target plane defined by the velocity at perigee.

Using the secular theory and quantifying the short-period component amplitude, we discard regions in the B-plane in which keyholes cannot exist and point out where numerical integration is needed to find keyholes.

We study the cases of common NEOs and observe when and how the predictions of the analytical theories correctly match the trajectories obtained with the higher fidelity numerical integrations. In some cases we find that the secular theory correctly predicts the MOID over time, such as the variation in the MOID along the resonant circle in Figure 2. The same technique is applied to other regions of the B-plane so we can compare the keyholes obtained with the different methods, and outline the limits of applicability of the secular theory.

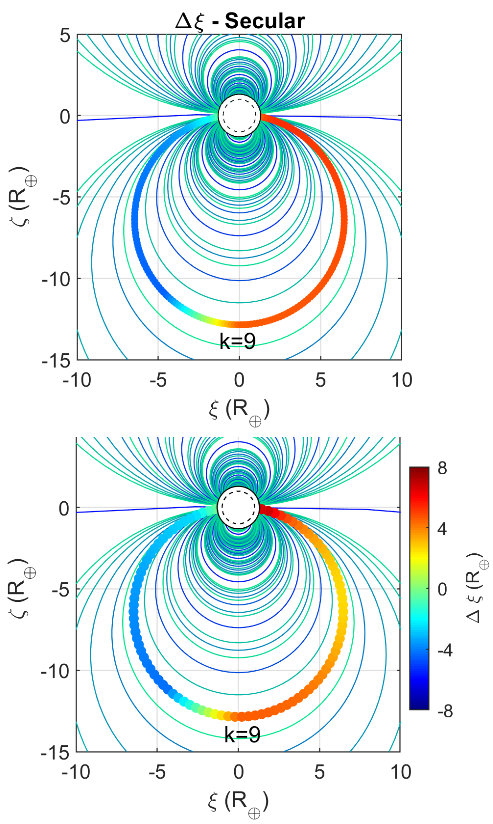

Figure 2. MOID variation (in Earth radii $R_\oplus$) at the resonant encounter around the resonant circle of an encounter after 9 years using: secular propagation as perturbed by Jupiter using the Lagrange-Laplace solution (top) and by numerical integration with the 8 planets as third-body perturbers (bottom) for 2021 PDC asteroid.

Acknowledgements: This research has made use of data and/or services provided by the International Astronomical Union's Minor Planet Center. This research is funded by the Balsells Fellowship Program at the University of Colorado Boulder and by a grant from the California Institute of Technology / Jet Propulsion Laboratory. The authors acknowledge the financial support offered by the *la Caixa* Foundation (ID 100010434) under agreement LCF/BQ/AA20/11820034 to support the PhD studies of A.P.F.

References:

[1] A. ~B. Chamberlin *et al.*, "Sentry: An Automated Close Approach Monitoring System for Near-Earth Objects," in *AAS/Division for Planetary Sciences Meeting Abstracts #33*, 2001, vol. 33, p. 41.08.

[2] A. Milani, S. R. Chesley, M. E. Sansaturio, G. Tommei, and G. B. Valsecchi, "Nonlinear impact monitoring: Line of variation searches for impactors," *Icarus*, 2005.

[3] J. Roa, D. Farnocchia, and S. R. Chesley, "A Novel Approach to Asteroid Impact Monitoring," *Astron. J.*, vol. 162, no. 6, p. 277, Dec. 2021.

[4] D. Vokrouhlický *et al.*, "The Yarkovsky effect for 99942 Apophis," *Icarus*, 2015.

[5] E. J. Öpik, *Interplanetary encounters: close-range gravitational interactions*. Elsevier Scientific Pub. Co., 1976.

[6] G. B. Valsecchi, E. M. Alessi, and A. Rossi, "An analytical solution for the swing-by problem," *Celest. Mech. Dyn. Astron.*, vol. 123, pp. 151–166, 2015.

[7] G. B. Valsecchi, A. Milani, G. F. Gronchi, and S. R. Chesley, "Resonant returns to close approaches: Analytical theory," *Astron. Astrophys.*, vol. 408, no. Number 3, September IV 2003,
pp. 1179–1196, 2003.

[8] C. D. Murray and S. F. Dermott, *Solar System Dynamics*. 2000.