# Dynamic Finite Element Modeling Approach for the Dynamic Stress Evolution of 99942 Apophis during its 2029 Earth Encounter

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**Introduction:** 99942 Apophis is going to closely encounter the Earth on April 13, 2029 [1]. This asteroid, which may have been inserted into the current orbit from the inner main belt due to the $\nu_6$ resonance [2], has a high concentration of LL chondrite on its surface [2] and an equivalent diameter of ~0.34 km [3]. Because of the slow rotation with tumbling [4], Apophis is considered to have the spin state change during the encounter [5, 6].

A key question for Apophis's physical process during the Earth encounter is whether the asteroid experiences modification processes on the surface or in the interior. If such processes happen, their magnitudes are a critical parameter to assess the physical properties of this asteroid. Numerical studies have attempted to give insights into this issue. Surface layers may be modified at a small level during the encounter [7], while tidally driven acceleration may cause a deformation of < 0.2 mm [8]. Here, we introduce a dynamic finite element model (DFEM) approach to analyze the time-evolution of deformation and stress in an irregularly shaped body and apply it to the case of Apophis. We hope that our simulation technique can tie-up with the existing state-of-the-art numerical methods to shed light on Apophis' physical processes during its 2029 Earth encounter.

**Dynamic FEM (DFEM) approach:** We have long been applying an FEM approach that focuses on static problems (i.e., no time evolution). This approach encountered several issues. First, it was necessary to assume that loadings were constant regardless of dynamic processes. Second, the applications of this case were limited. Certainly, no applications can be made to dynamic processes because such processes are affected by both loadings and deformation. To solve these issues, we have developed a simulation package written in the C++ platform.

The DFEM approach accounts for (1) the orbital and attitude motion and (2) the deformation process. In the current version of the package, the deformation is assumed to be small, and the orbital and attitude motion is decoupled from the deformation process. Therefore, the program first computes the orbital and attitude conditions of the body and then determines the stress distribution by using both loadings at a given time and the stress state at the previous time. The present version considers elastic deformation.

**Boundary conditions:** It was necessary for our static FEM approach to constrain six degrees of freedom to mimic the environment where an asteroid rotates without any fixed points. Because of the nature of FEM, for calculations of the equilibrium condition, zero boundary conditions lead to singularity problems and thus no solutions. We developed a function that can distribute mass elements properly and remove the rigid motion in the body-fixed frame. Using this function, we reduced the number of node constraints to three degrees of freedom, allowing for avoiding unrealistic stress concentrations more robustly than our static FEM approach.

**Initial conditions:** The initial conditions are defined to represent the structural variations during a dynamic process. If the deformation (and thus stress) is set to be zero originally, dynamic simulations do not give solutions properly and even make them diverge. In our simulation package, the initial condition is automatically determined by considering the equilibrium condition; the time-evolution term is set to be zero to determine the deformation and stress. The dynamic simulation then uses this condition as the initial conditions of the state to be solved.

**Applications to Apophis:** We apply the DFEM approach to Apophis. We decide to use the radar-driven shape model developed by [3], while we are aware that the accuracy of this model may be similar to the light curve-driven shape model developed by [4]. Therefore, the shape model of Apophis still needs to be updated by detailed observations in the future.

To implement the shape model to the DFEM approach, we first reduce the resolution to save the simulation burden and then develop a 4-node tetrahedron FEM mesh. This process leads to 673 nodes and 2350 elements. We note that the coarse resolution averages the detailed stress distribution over a large-element volume; however, our focus here is on describing the stress distribution on a global scale. This reduction process still gives meaningful results.

**Simulation settings:** Our simulation conditions are described in Table 1. We confirm that the 0.02-sec step size case is consistent with cases with shorter step sizes. While an earlier numerical model study assumed the bulk density to be 2,900 kg/m$^3$ [8], we define it as 2,000 kg/m$^3$. This setting is based on Itokawa's bulk density and the recent observation study arguing the existence of LL chondrite [2]. However, the correlation between Itokawa's material compositions and LL chondrite is still open to discussion. We also ignore the tumbling mode, which should be negligible because the principal rotation is also too long to affect the stress component. Also, the initial time is set to be ~4 h before the encounter; therefore, the tidal effect is negligible at the initial condition.

Table 1. Simulation settings. Initial date and time are derived from JPR Horizons web-interface.

| Parameters | Values | Units |
|---|---|---|
| **Initial date** | April 13, 2029 | [-] |
| **Initial time** | 18:00 (TDB) | [h] |
| **Simulation time** | 8 | [h] |
| **Step size** | 0.02 | [sec] |
| **Bulk density** | 2,000 | [kg/m3] |
| **Principal rotation** | 30.56 | [h] |
| **Young's modules** | $1 \times 10^7$ | [Pa] |
| **Poisson's ratio** | 0.25 | [-] |

**Results:** We first discuss the initial stress distribution in Apophis. Figure 1 shows the pressure distribution on the surface. The radar model exhibits a bifurcation feature. The neck region (lower middle) is more compressed than other areas due to the moment mainly driven by self-gravity, having a pressure of around -10 Pa. Pressure in the major regions is negative, meaning that they experience compression. However, some limited areas reach tensile conditions (< ~2 Pa). Such tensile conditions are likely to be influenced by local topography.

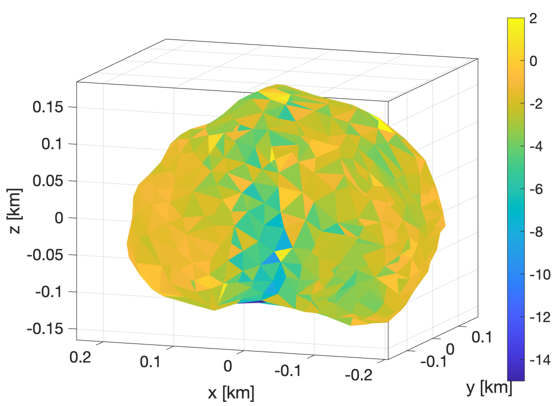


Figure 1. Pressure distribution on the surface of Apophis at the initial condition. Pressure is described as a negative value.

Next, we introduce the variation in pressure during the tidal encounter. The present version of the DFEM approach assumes the elastic mode and thus does not have a time-decay term for deformation. Therefore, we observe high-frequency variations in the deformation and stress distributions. We remove this high-frequency response by employing polynomial fitting and compute the stress variations. We confirm that the amplitude of the high-frequency mode is much smaller than the observed variations due to the tidal effect.

Figure 2 shows the maximum pressure variations when Apophis approaches the closest point to Earth. The variations are defined as the pressure at a given time to the initial pressure. We find that the neck region experiences reduced pressure (again, the pressure is defined to be negative). This feature results from the fact that during the close encounter, the body is stretched out along the longest axis. The pressure variation is about $\pm 0.2$ Pa.

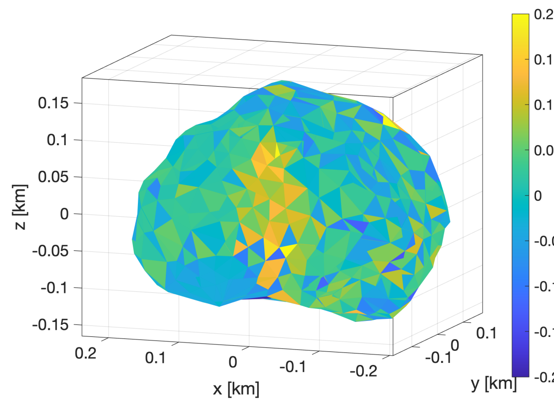


Figure 2. Maximum pressure variations during the close encounter.

**Discussions:** Our simulation gave reasonable stress conditions and variations when Apophis approaches Earth. We interpret our results in the following ways. First, the structural condition in a large element may not change significantly. As shown in Figure 2, a possible variation in pressure may be up to ~0.2 Pa. Second, if there is a bifurcation as the radar model predicts, it is likely that the neck region may be subject to such limited variations. The interior at a large scale may not have physical modifications during the Earth encounter. These results are consistent with findings from earlier numerical work [8]. We note that our results are preliminary, and DFEM simulations at high resolution with accurate shape models will be able to give more detailed structural conditions at local scales.

**References:** [1] Farnocchia et al. (2013), Icarus, 224, 192-200. [2] Reddy et al. (2018), AJ, 155:140. [3] Brozović et al. (2018), Icarus, 300, 115-128. [4] Pravec et al. (2014), Icarus, 233, 48-50. [5] Scheeres et al. (2005), Icarus, 178, 281-283. [6] Souchay et al. (2018), A&A 617, A74. [7] Yu et al. (2014), Icarus, 242, 82-96. [8] DeMartini et al. (2019), Icarus 328, 93-103.