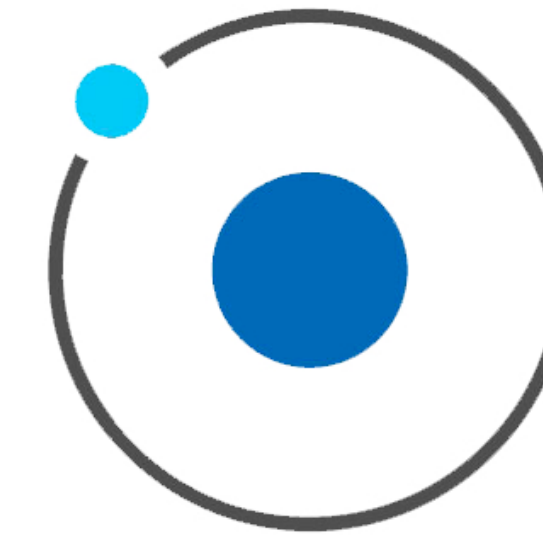


# An estimation of the Yarkovsky effect for (99942) Apophis via high-order Taylor polynomials

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- Using `TaylorIntegration.jl`<sup>1</sup>, we propagate the orbit of Apophis as a Taylor polynomial w.r.t. time  $t$ , changes in the initial position/velocity  $\delta\mathbf{x}_0$  and the Yarkovsky parameter  $A_2$ . This technique is known as *jet transport*.
- From this, we compute the (weighted) mean square residual  $Q(\delta x_0, A_2) = \chi^2/n_{\text{obs}}$  as a Taylor polynomial in the unknown orbital parameters. We use all available radar astrometry for this object (17 delay, 29 Dopplers) and selected optical astrometry (472 RA/Dec pairs) following Vokrouhlický et al. (2015).
- We obtain two orbital fits to radar+optical data: a gravity only 6-DOF (**OR6**) and a 7-DOF fit (**OR7**) which accounts for Yarkovsky. We find local minima of the Taylor expansion of  $Q$  via Newton method.
- We find a mean semi major axis drift  $\langle \dot{a} \rangle = (-341 \pm 158) \text{ m/yr}$ , corresponding to  $A_2 = (-5.0 \pm 2.8) \times 10^{-14} \text{ au/d}^2$ .

<sup>1</sup> URL: [github.com/PerezHz/TaylorIntegration.jl](https://github.com/PerezHz/TaylorIntegration.jl)

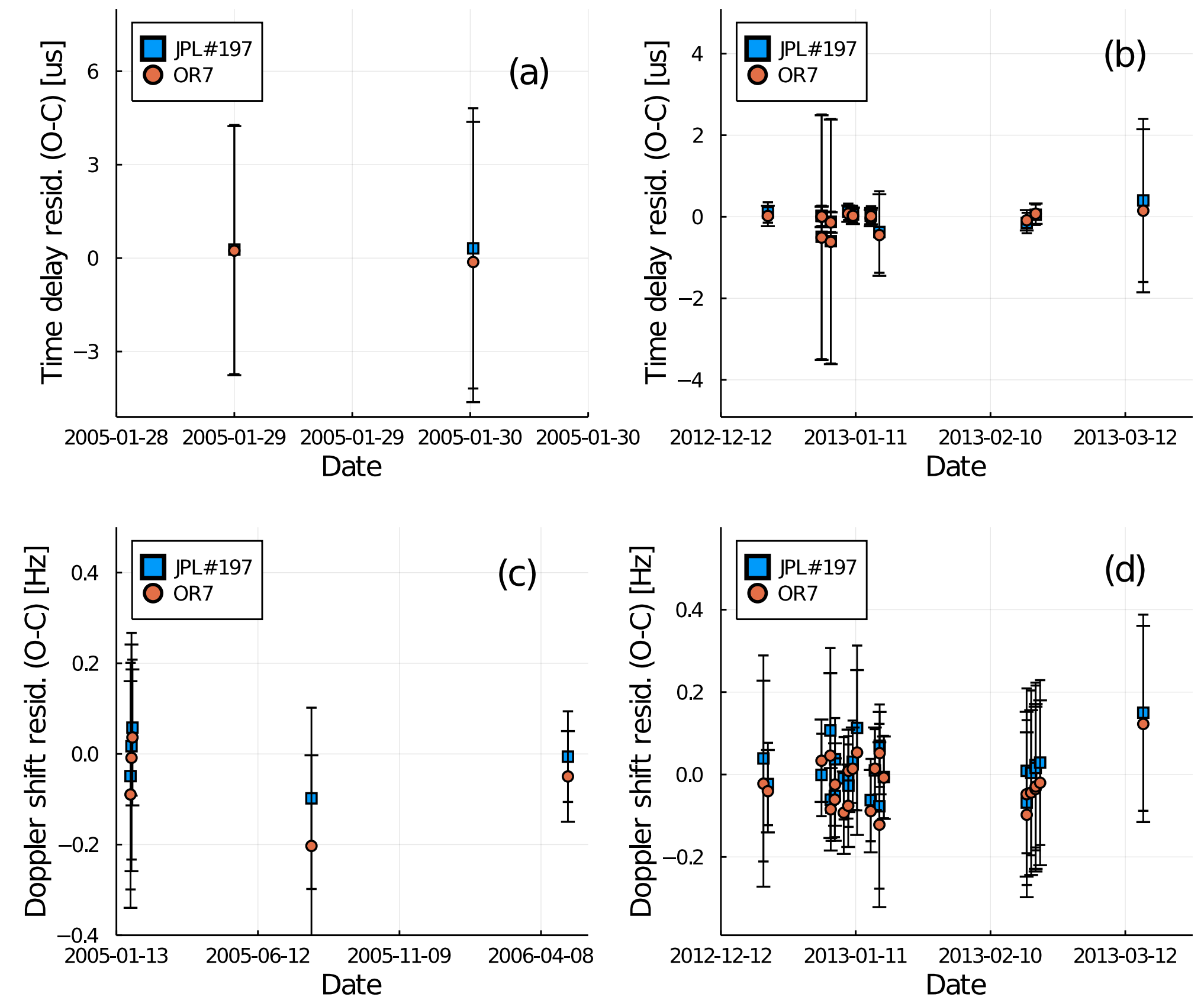
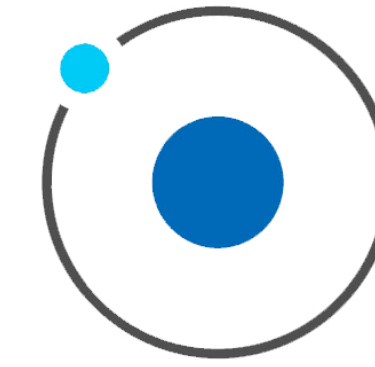
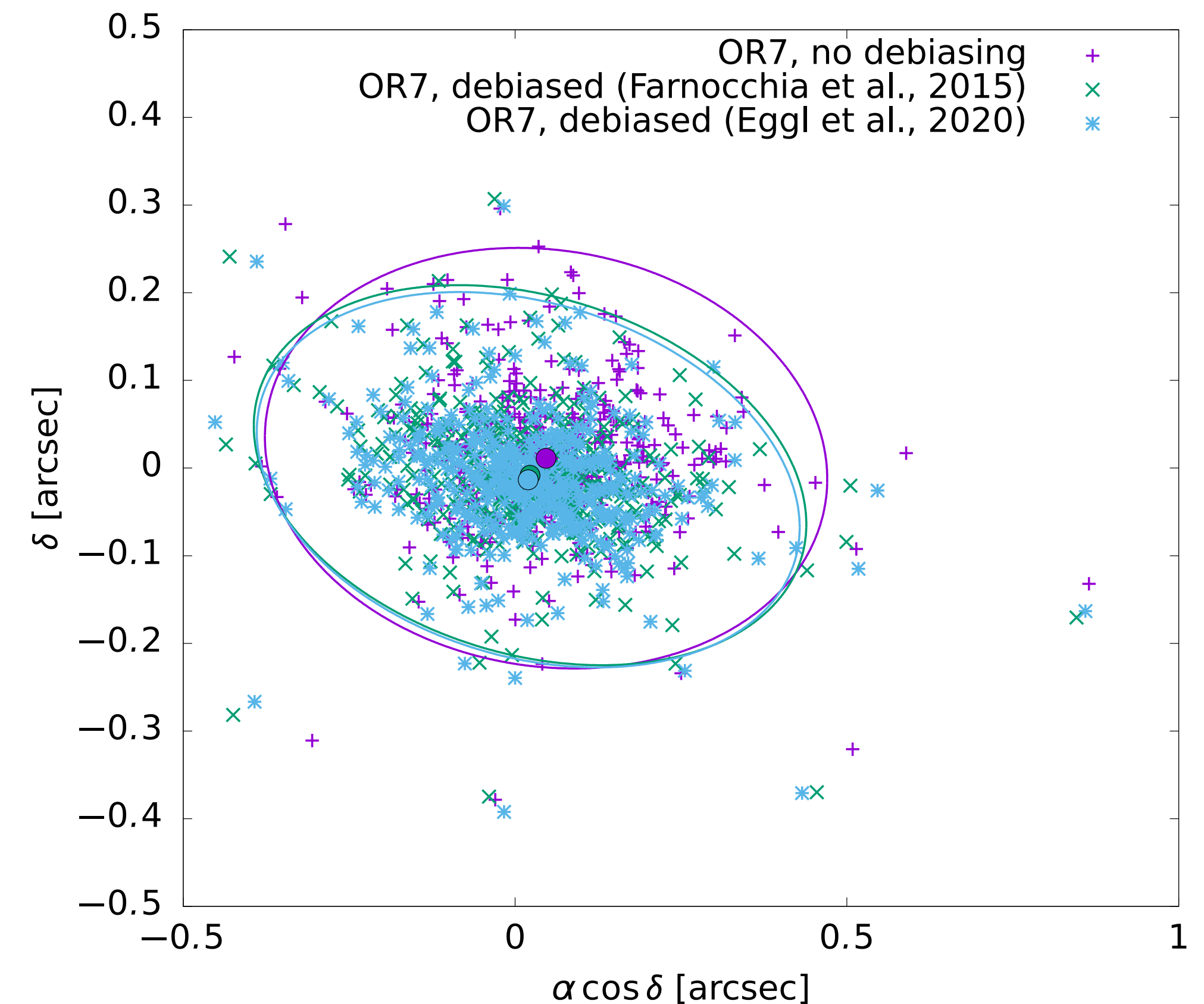


Fig: Post-fit delay/Doppler astrometric residuals (OR7).



## Optical astrometry postfit residuals

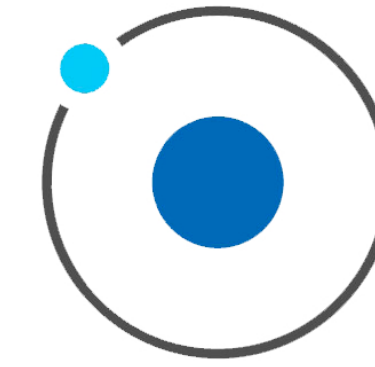
- Our optical astrometry error model accounts for biases present in star catalogs, and accounts for other sources of systematic errors via an appropriate weighting scheme.
- We compare the post-fit astrometric residuals for observations from Tholen et al. (2013), for two debiasing tables: Farnocchia et al. (2015) and Eggl et al. (2020).
- Using Eggl. et al. (2020) debiasing table, we find a mean RA/DEC postfit residual  $(0.011'', -0.009'')$  for the Tholen et al. (2013) astrometry.



**Fig:** Post-fit residuals for Tholen et al. (2013) optical astrometry. Ellipses correspond to  $3\text{-}\sigma$  level.

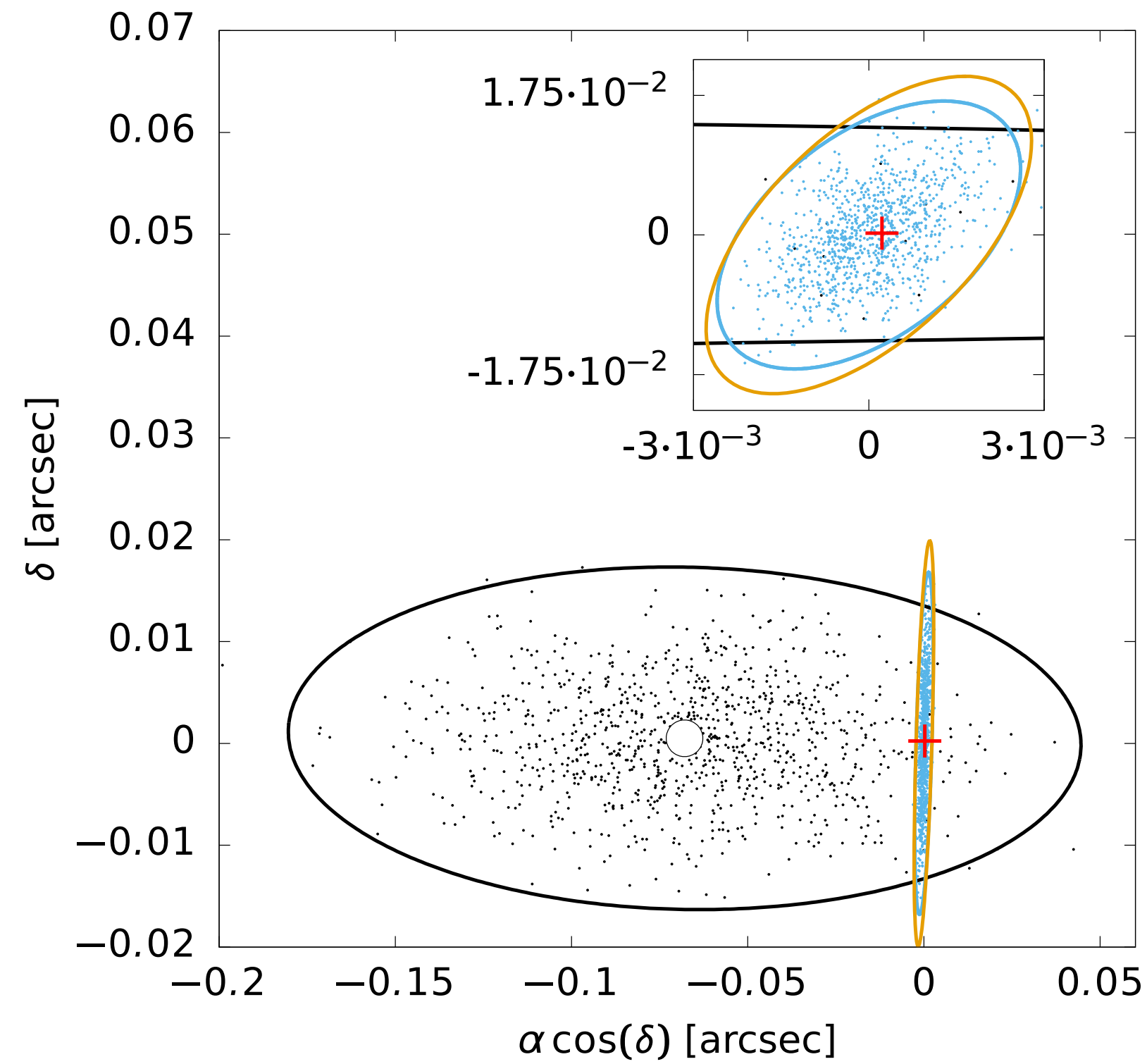
# An Estimation of the Yarkovsky Effect on Asteroid (99942) Apophis via High-Order Taylor Polynomials

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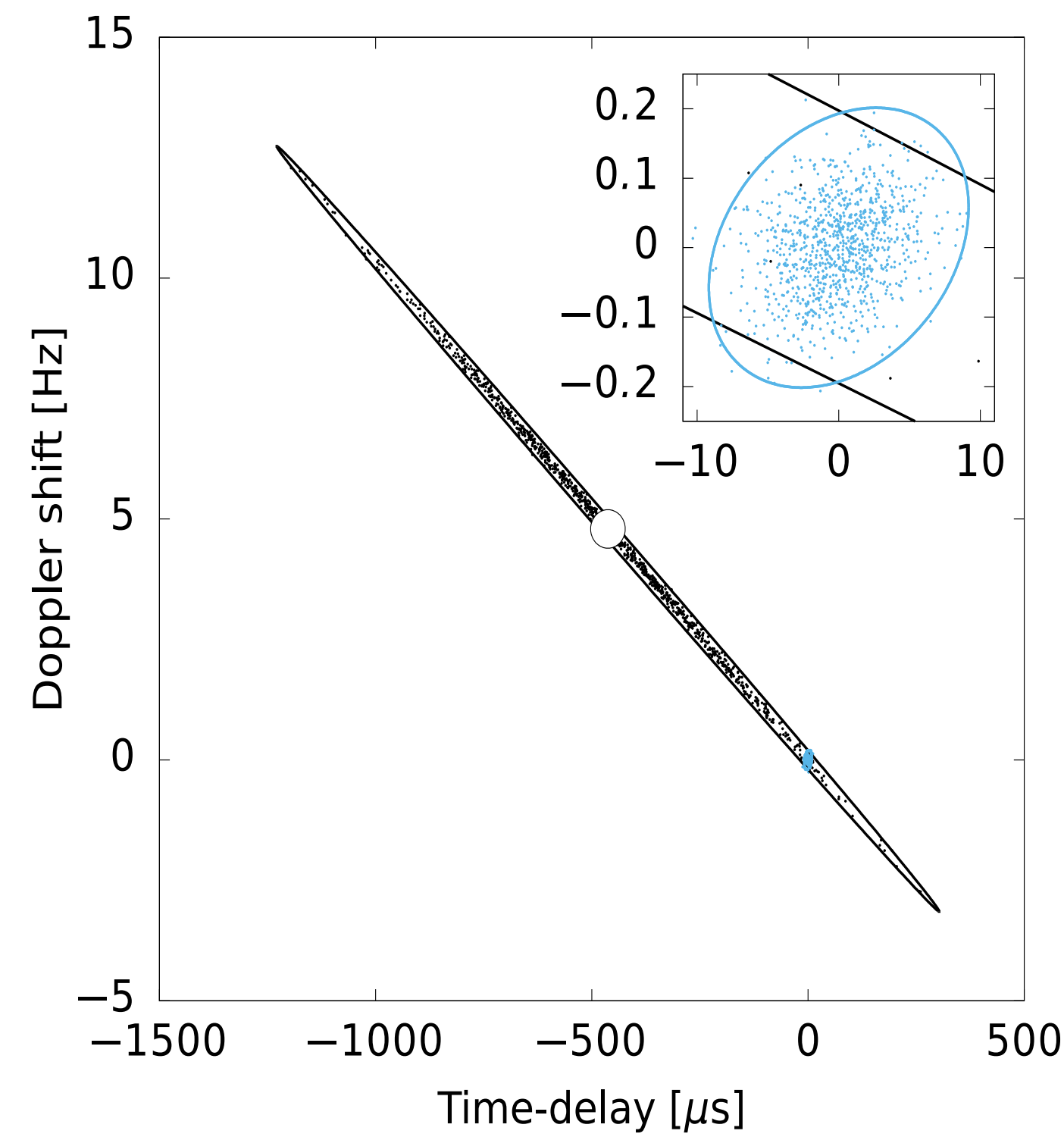
## Optical astrometry residuals: OR6, OR7 prediction vs observed value (Mt. Lemmon G96, 2020-Jun-08)



$$\alpha_{OR6} = 143.806 \text{ deg}$$

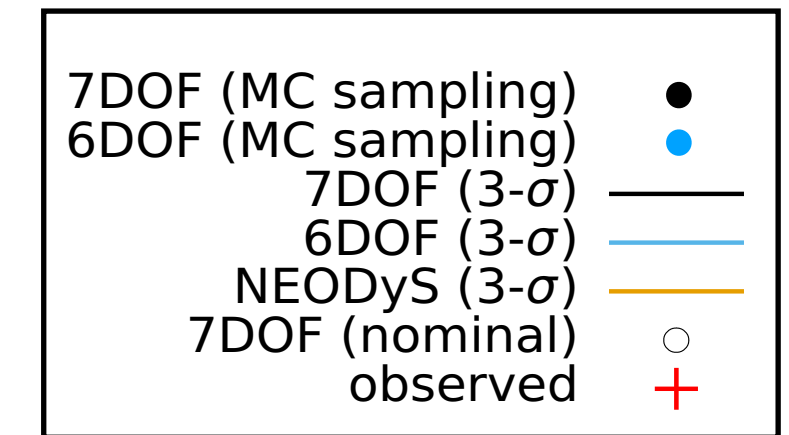
$$\delta_{OR6} = 14.646 \text{ deg}$$

## Radar astrometry prediction: OR6 vs OR7. (Goldstone, 2021- Mar-06 11:00 UTC)

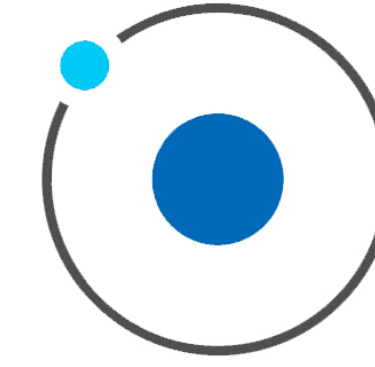


$$\tau_{OR6} = 112,427,832.2 \mu\text{s}$$

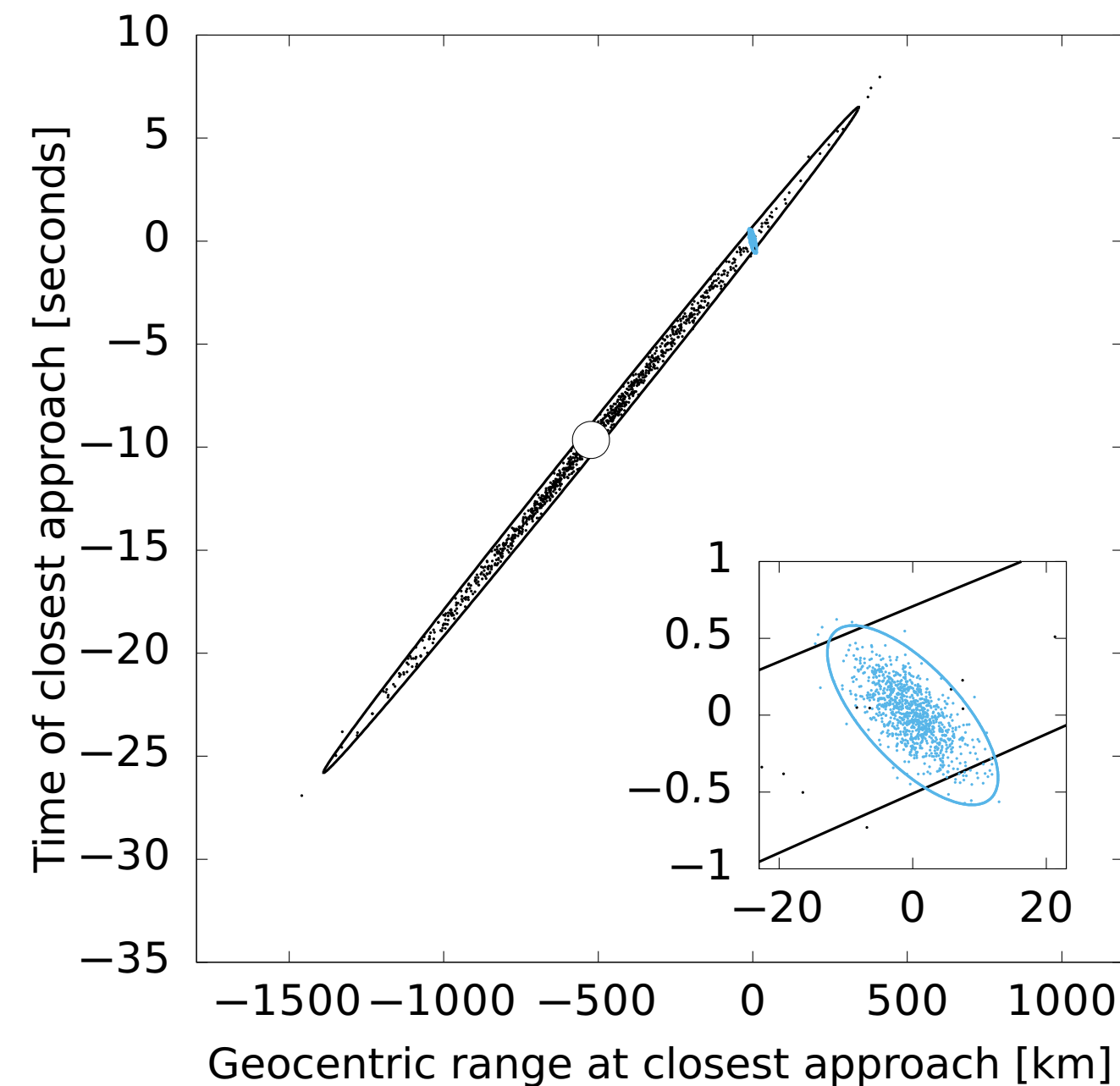
$$\nu_{OR6} = -24485.179 \text{ Hz}$$



- OR6: gravity-only 6-DOF orbital fit to radar+optical
- OR7: non-grav. 7-DOF orbital fit to radar+optical



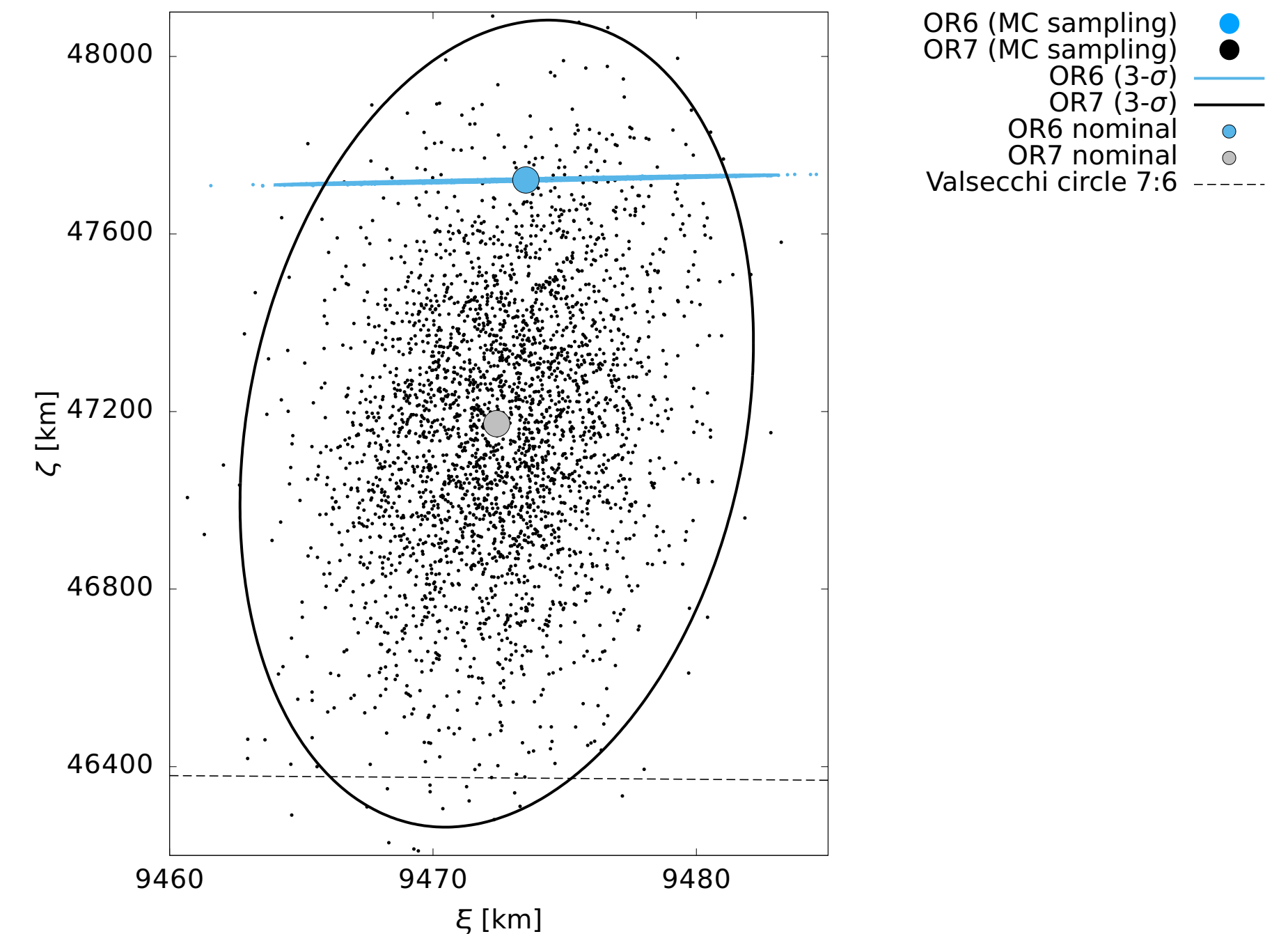
## 2029 close approach: OR6 vs OR7



OR7 (MC sampling) ●  
OR6 (MC sampling) ●  
OR7 3- $\sigma$  —  
OR6 3- $\sigma$  —  
OR7 nominal ○

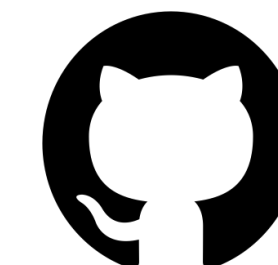
$$d_{\text{OR6}}^* = 38,314 \text{ km}$$
$$t_{\text{OR6}}^* = 21:46:18.475 \text{ TDB}$$

## 2029 B-plane (Öpik's frame)



### Conclusions:

- We implemented a software package for NEA orbit determination and uncertainty propagation based on high-order Taylor method. To be made public soon; stay tuned!
- We provide an independent (marginal) estimation of Yarkovsky effect for Apophis from data, consistent with previous results.
- WIP: updating orbital fits with 2019-2020 data.

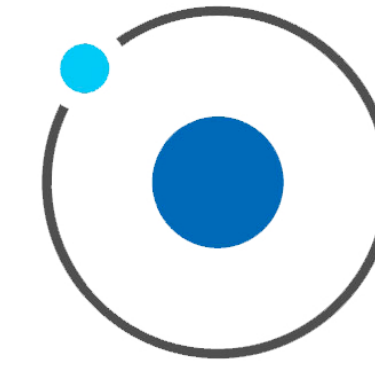


**GitHub:**

**@PerezHz**

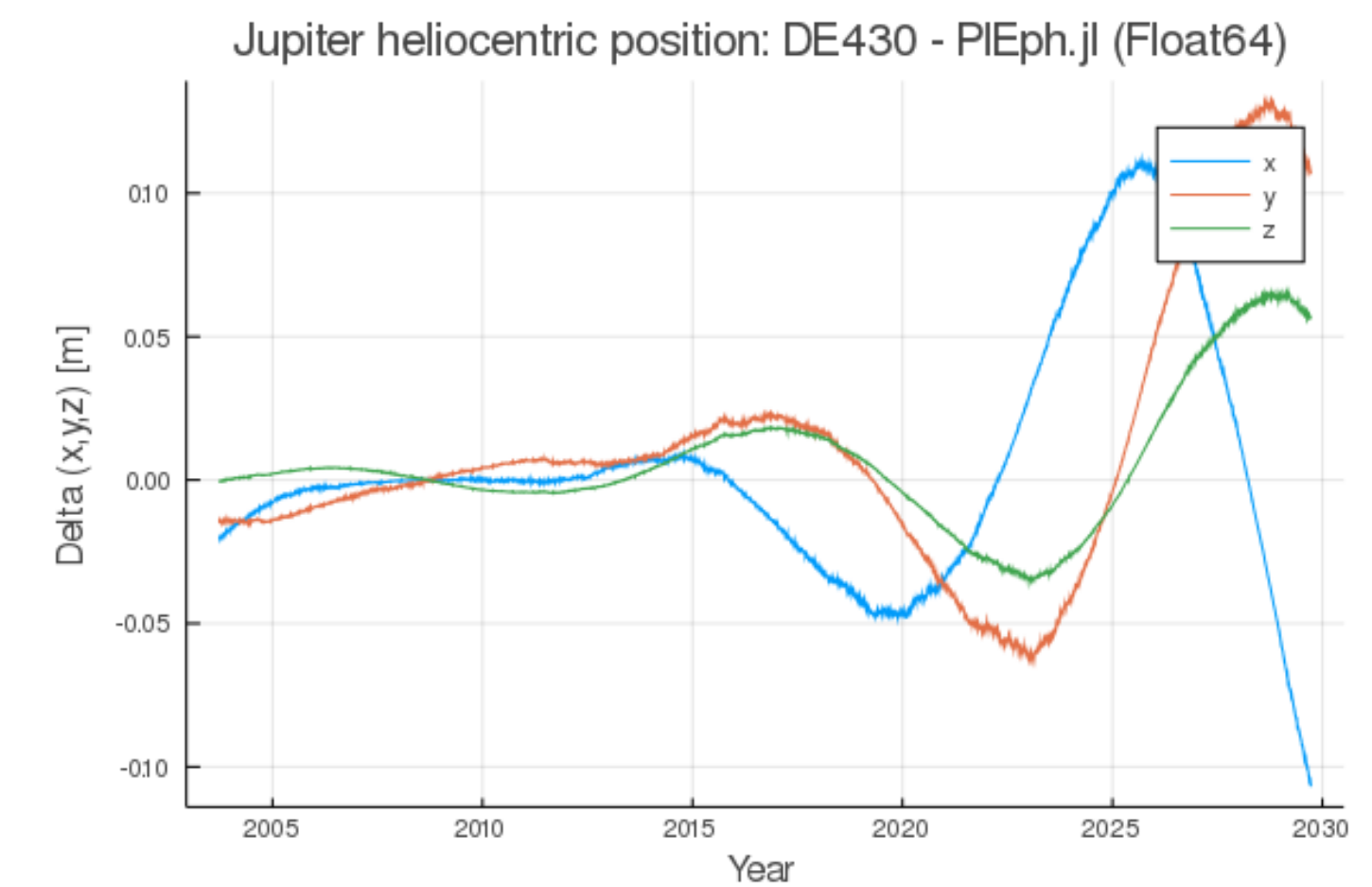
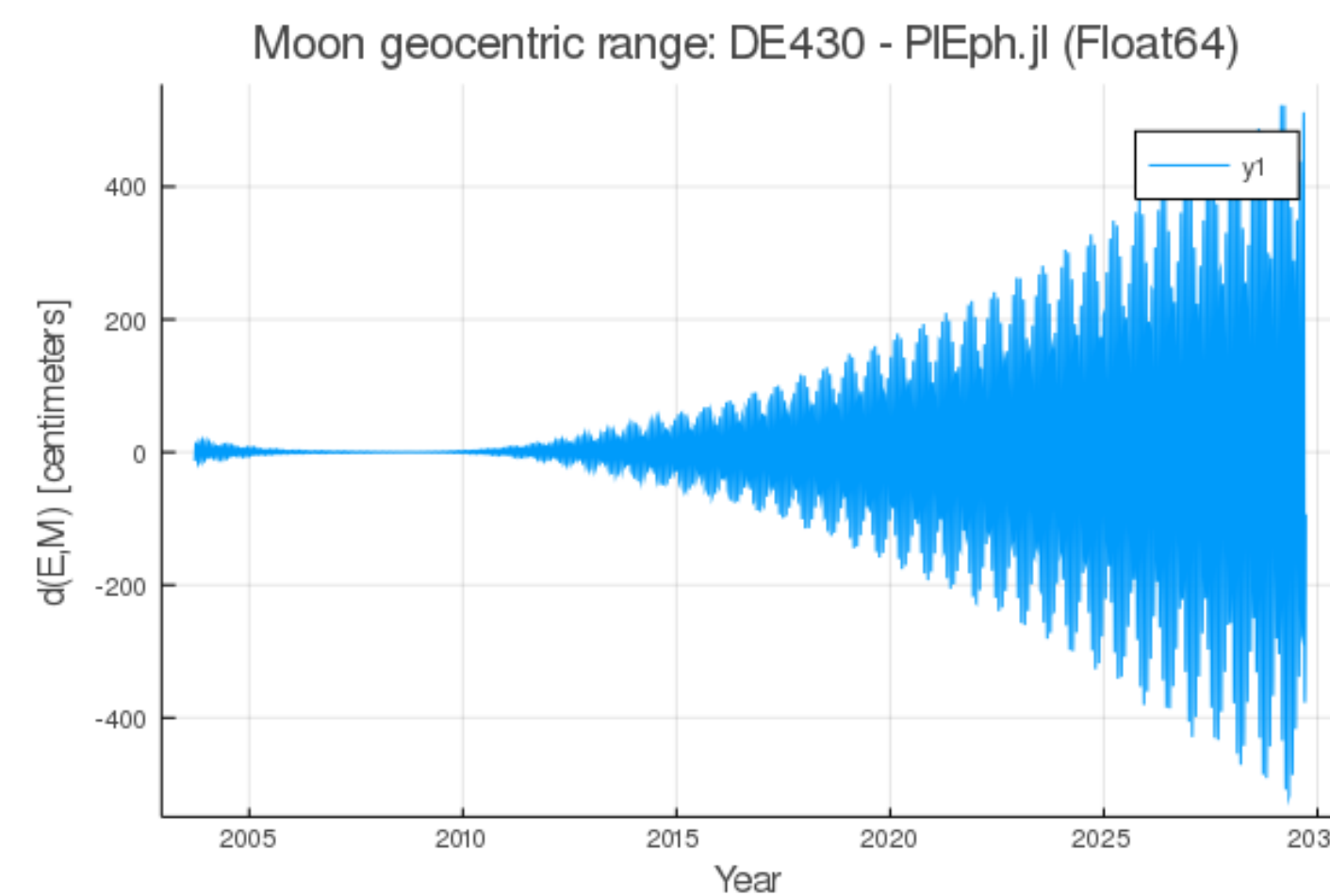
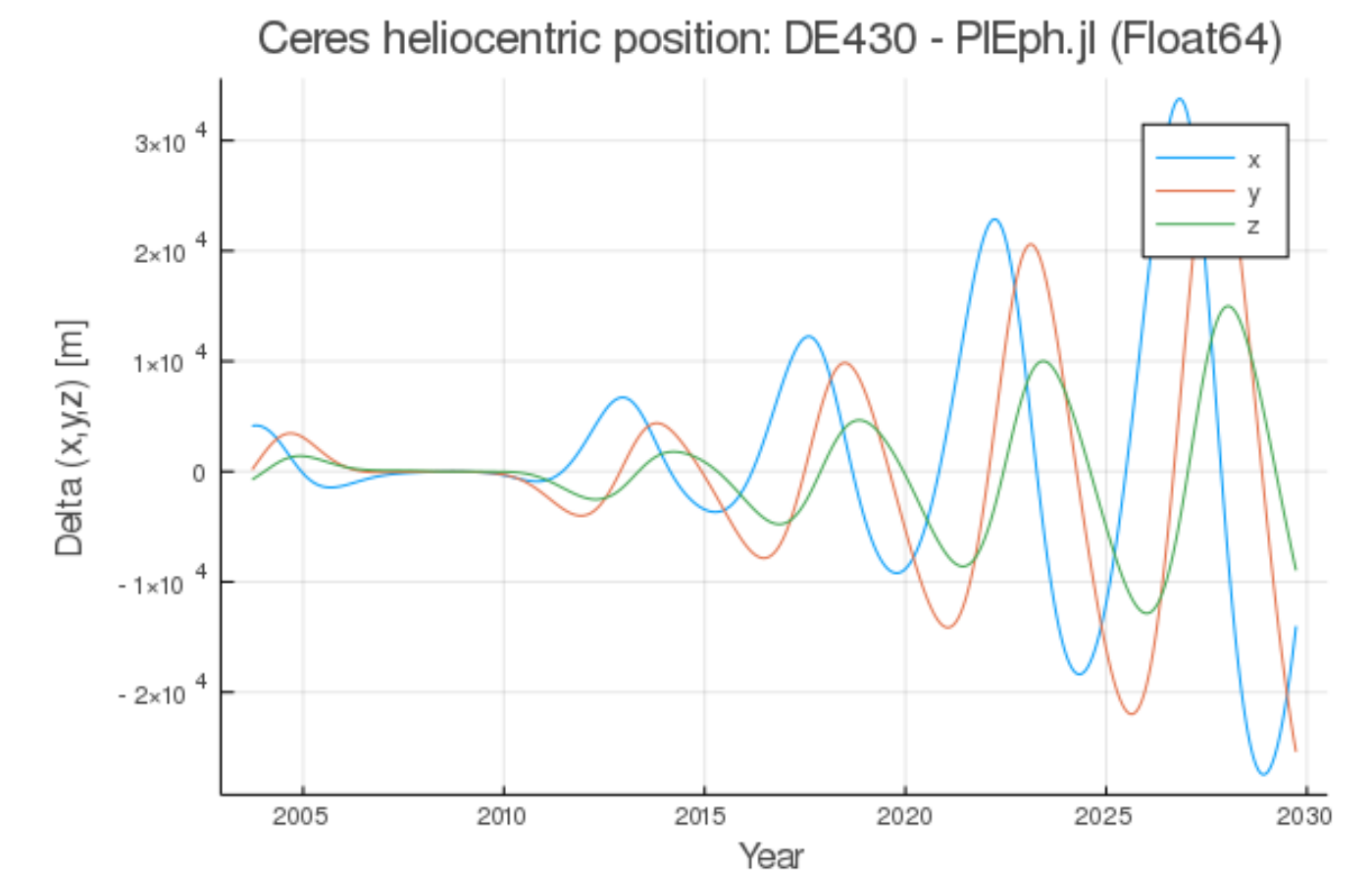
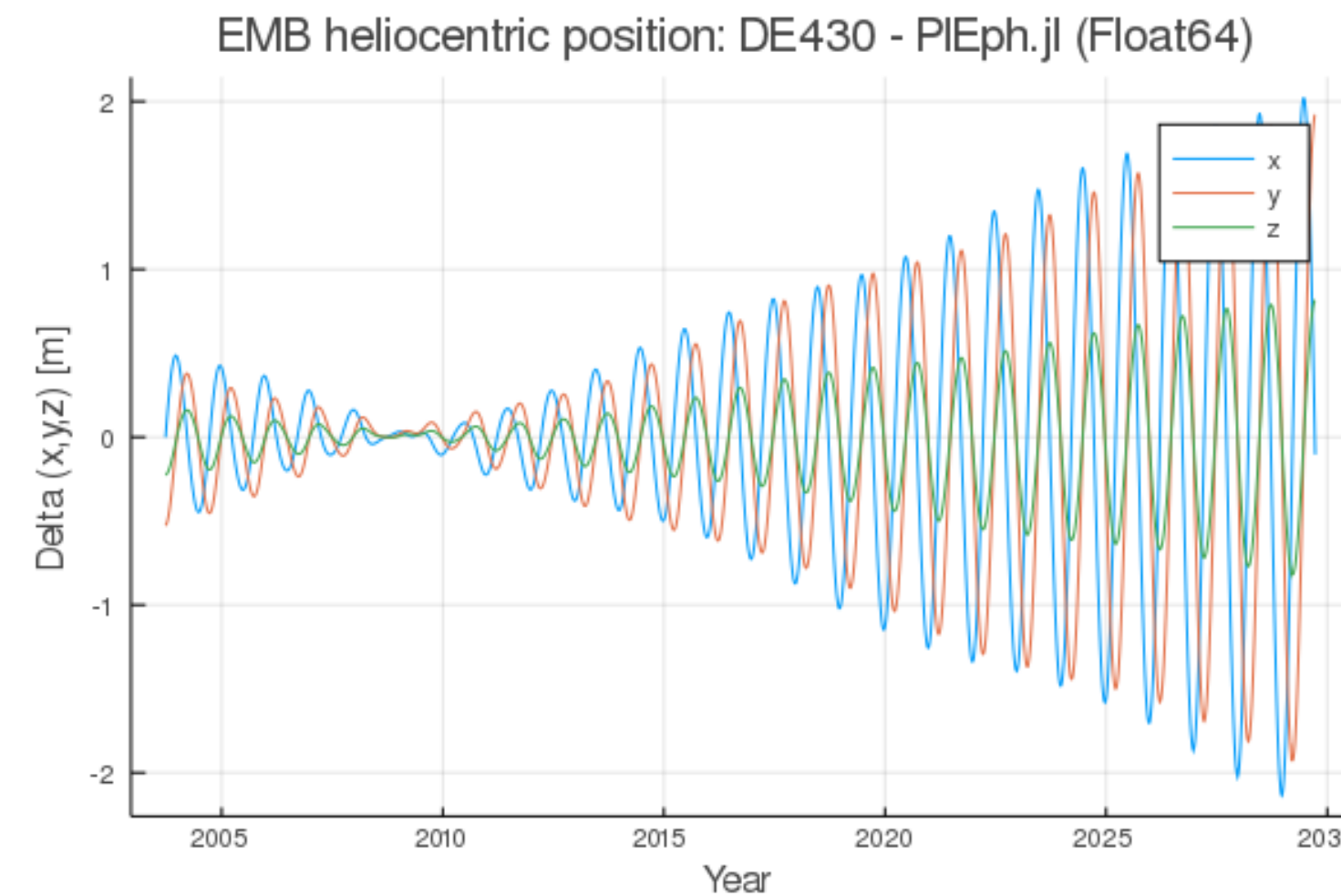
**[PerezHz/TaylorIntegration.jl](https://github.com/PerezHz/TaylorIntegration.jl)**

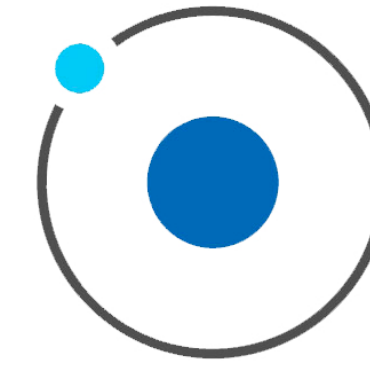
**Acknowledgments:** We acknowledge funding support provided by PAPIIT-UNAM IG100819 and computing resources provided by LANCAD-UNAM-DGTIC-284.



## Planetary ephemeris

- To integrate the orbit of Apophis, as planetary ephemeris we use our own numerical integration of the JPL DE430 dynamical model (Folkner et al., 2014).
- In the time-span of our numerical integrations (2004-2029), we are able to reproduce the planetary heliocentric positions and lunar geocentric range below the ~5 m level.





## Apophis orbital motion model

Our dynamical model for Apophis orbital motion includes:

- Post-Newtonian interactions with the Sun, the eight planets, the Moon and Pluto.
- Newtonian interactions with 16 most-massive asteroids.
- Acceleration due to Earth's  $J_2$ .
- We model the Yarkovsky effect as an acceleration in the transverse direction:  
 $\mathbf{a}_t = A_2/r^2 \hat{\mathbf{t}}$ .
- We integrate the equations of motion exploiting jet transport techniques, as implemented in `TaylorIntegration.jl`. Jet transport allows us to compute the  $q$ -th order Taylor expansion of the orbit around a given set of orbital parameters  $\mathbf{x}_0$ :

$$\tilde{\mathbf{x}}(\delta\mathbf{x}_0, t) = \mathbf{x}^{[0]}(t) + \sum_{l=1}^q \mathbf{x}^{[l]}(t)(\delta\mathbf{x}_0)^l.$$