The role of the major planets in the libration of Pluto's argument of perihelion

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The longitudinal libration of Pluto's longitude of perihelion, ~90˚away from Neptune, has long been understood – it is a consequence of its 3:2 mean motion resonance with that planet. On the other hand, the libration of Pluto's argument of perihelion, *g*, is less well understood. We investigate the role of the resonant perturbations from Neptune and the role of the secular perturbations of Jupiter, Saturn and Uranus, in maintaining Pluto's *g* libration and long-term stability.

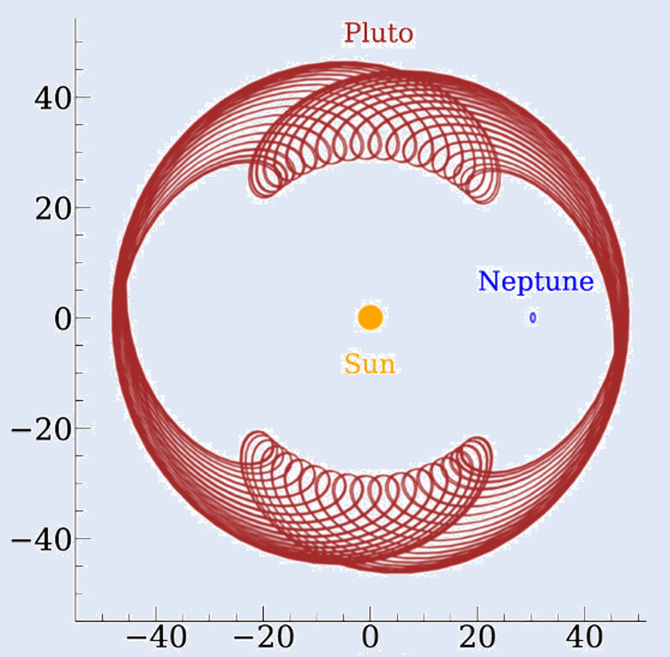
- A key parameter is $k^2 = (1-e^2)\cos^2 I$: Libration of *g* is possible only when $k^2 < k^2_{\rm crit}$
  - ✓ In the circular restricted 3-body system of Sun-Neptune-(non-resonant) TNO, $k^2_{\rm crit}$ lies in the range ~0.2−0.3 ($a_{\rm N}/a_{\rm TNO} < 1$)
  - ✓ For Plutinos ($a_{\rm N}/a_{\rm TNO} \sim 0.76$): including Neptune's 3:2 resonant effects, $k^2_{\rm crit}$ is much larger, in the range ~0.82−0.88; when we also include the secular effects of Jupiter, Saturn and Uranus, then $k^2_{\rm crit}$ lies in the even larger but narrow range of ~0.93−0.94
- Key take-away: Pluto's (and Plutinos') *g* libration and its long-term stability is substantially dependent on the orbital architecture of all four giant planets, not just that of Neptune

Characteristics of Pluto's perihelion

(1) It stays ~90˚ away from Neptune's longitude
- a consequence of its 3:2 mean motion resonance with Neptune
- Critical resonant argument σ librates
  $\sigma = 3\,\lambda_{\rm Pluto} - 2\,\lambda_{\rm Neptune} - \varpi_{\rm Pluto} \sim 180°$
- Called "azimuthal libration"

(2) Its argument of perihelion, *g*, stays around ~90˚ from the node
- Called "latitudinal libration"

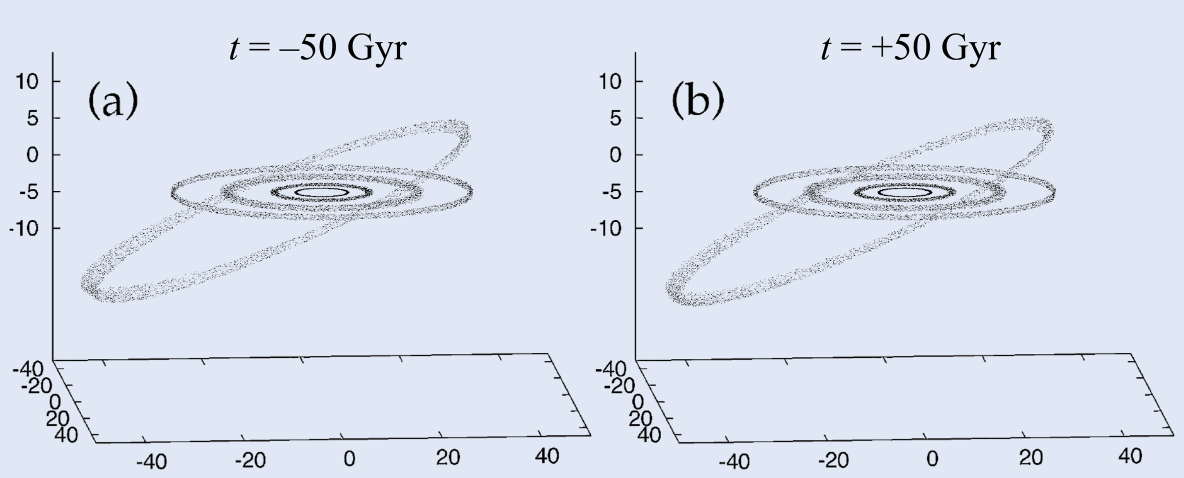
✓ Our question here – What maintains the *g* libration of Pluto (or Plutinos)?



Zaveri and Malhotra (2021)

These two characteristics remain stable for very long time, *O*(10^9) years and more

1.814×10^9 yr around →



Note: Pluto's perihelion is fixed along the *x*-direction

Ito & Tanikawa (2002)

Summary

- We tested several models to identify the influence of the other giant planets (Jupiter, Saturn and Uranus) on the secular motion of Pluto's (and Plutinos') argument of perihelion *g*.
- We found that the model with Sun's J_2 as a proxy for the secular effects of Jupiter, Saturn, Uranus, together with a sinusoid model for libration of the critical resonant argument σ explains the secular behavior of Pluto and Plutinos quite well.
- Consideration of more subtle and complicated effects such as Neptune's eccentricity or the 2:1 mean motion "near resonance" between Uranus and Neptune is not necessary for this purpose.

Method – numerical quadrature

$$\frac{d^2\boldsymbol{r}}{dt^2} + \mu\frac{\boldsymbol{r}}{r^3} = \nabla R$$

Instead of integrating the full equations of motion (i.e. numerical orbit propagation), we average the disturbing function R and reduce its degrees of freedom

Non resonant system
$$\overline{R}(g,e) = \frac{\mu'}{4\pi^2}\int_0^{2\pi}\int_0^{2\pi}\frac{d\lambda d\lambda'}{\Delta}$$

Resonant system
$$\overline{R} = \frac{\mu'}{4\pi^2 p}\int_0^{2\pi p} R\, d\lambda'$$

In the mean motion resonant system, we keep the relationship between mean longitudes of perturbed body (λ) and perturbing body (λ′) through σ

$$\sigma = p\lambda + p'\lambda' - (p+p')\varpi$$

$p = 3, p' = -2$ for Pluto (and Plutinos)

Sinusoid model for the critical resonant argument σ

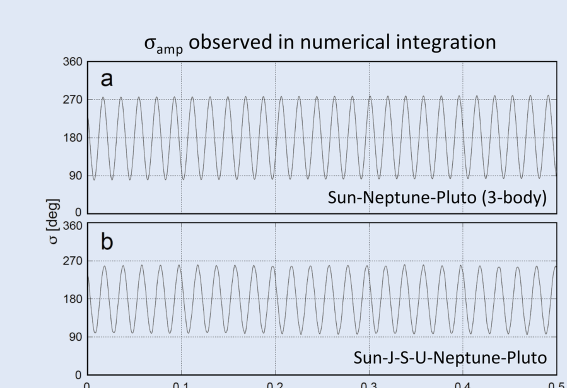
We approximate σ with a sinusoid (with $\sigma_{\rm amp}$ as a free parameter)

$$\sigma(\tau) = \sigma_{\rm amp}\sin\tau + \sigma_0$$

($\sigma_0 = 180°$ for Plutinos)

The averaged disturbing function is written as

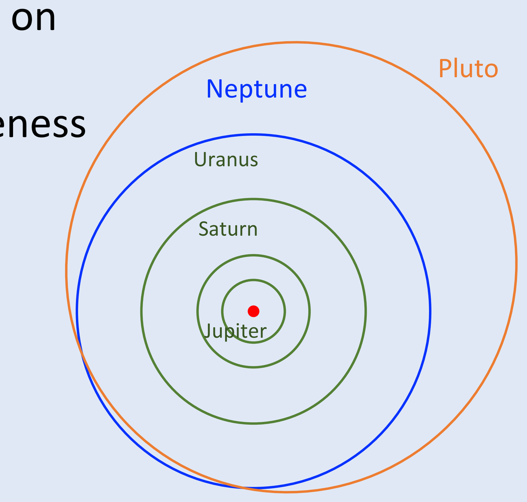
$$\overline{R} = \frac{\mu'}{4\pi^2 p}\int_0^{2\pi}\int_0^{2\pi p} R_{\sigma=\sigma(\tau)}\, d\lambda' d\tau$$



Quantification of planet's secular forcing using Sun's J_2

We approximate Jupiter, Saturn, Uranus as circular coplanar rings.
We tested two ways for this:
(a) Using Sun's effective J_2 as proxy for their secular effects
(b) Using higher-order Legendre polynomial expansion

We confirmed the results with numerical quadrature of the full equations of motion, and here mainly report on (a) - averaged (secular) perturbation from the inner three giant planets modeled with an effective oblateness of the Sun as follows.



Potential caused by an averaged ring

$$V_{\rm ring} = -\frac{Gm'}{r}\left[1 + \sum_{n=1}^{\infty}\left(\frac{a'}{r}\right)^{2n} P_{2n}(0) P_{2n}\left(\frac{z}{r}\right)\right]$$

Sun's non-sphericity causes the potential

$$V_{\odot} = -\frac{Gm_{\odot}}{r}\left[1 - \sum_{n=1}^{\infty} J_{2n}\left(\frac{R_{\odot}}{r}\right)^{2n} P_{2n}\left(\frac{z}{r}\right)\right]$$

Comparing the above expansions at the leading order, we obtain the "effective" J_2

$$J_{2,\rm eff} = \frac{1}{2}\frac{m_{\rm p} a_{\rm p}^2}{m_{\odot} R_{\odot}^2}$$

Values in the actual solar system

planet	$m_{\odot}/m_{\rm p}$	$a_{\rm p}$ (au)	$J_{2,\rm eff}$
Jupiter	1047.3486	5.2076	592.5
Saturn	3497.898	9.5725	605.5
Uranus	22902.98	19.3038	376.1
Total			1574.1

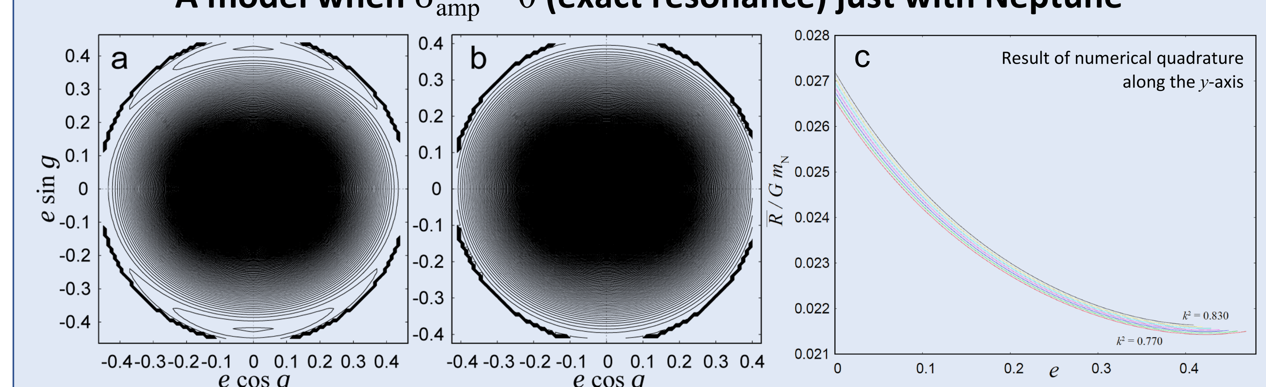
Malhotra & Ito (2022)

Drawing equi-potential contours with various k^2

After averaging, $k^2 = (1-e^2)\cos^2 I$ is a constant, and the disturbing function R becomes a function only of e and *g* (i.e. integrable).
We draw equi-R contours in the parameter plane ($e\cos g$, $e\sin g$).

Shape of the equi-R contours depends on k^2. If the contour map has a local minimum or maximum at certain k^2, there is a stationary point of e and *g*, and *g* librations can occur at this value of k^2 .

A model when $\sigma_{\rm amp} = 0$ (exact resonance) just with Neptune

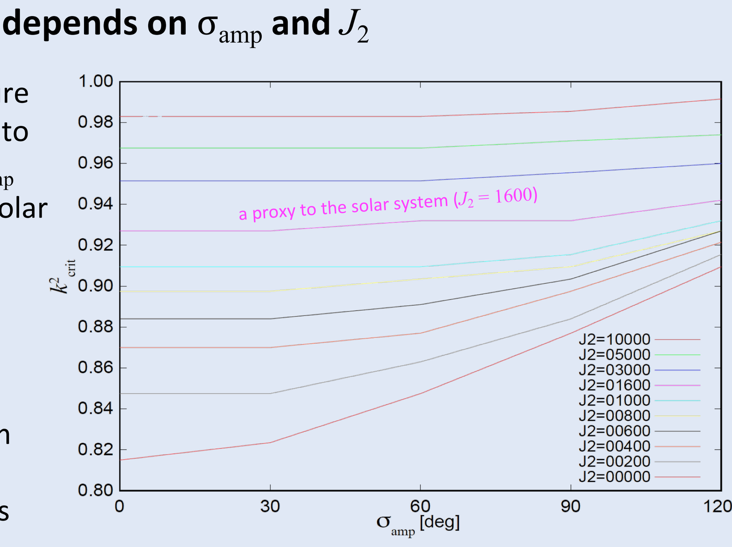


Equi-potential curves of a 3:2 resonant TNO ($a_{\rm N}/a_{\rm TNO} \sim 0.76$) drawn on the ($e\cos g$, $e\sin g$) plane. In panel (a; $k^2 = 0.785$), we find a pair of stationary points along the y-axis near $e \sim 0.4$ (a local minima of R). In the panel (b; $k^2 = 0.820$), we do not find any local extrema. So, $k^2_{\rm crit}$ lies somewhere between 0.785 and 0.820. By changing the value of k^2 with finer steps (panel c), we can determine $k^2_{\rm crit}$ more precisely: $k^2_{\rm crit} \sim 0.815$, or $I_{\rm crit} \sim 25.4°$.

$k^2_{\rm crit}$ depends on $\sigma_{\rm amp}$ and J_2

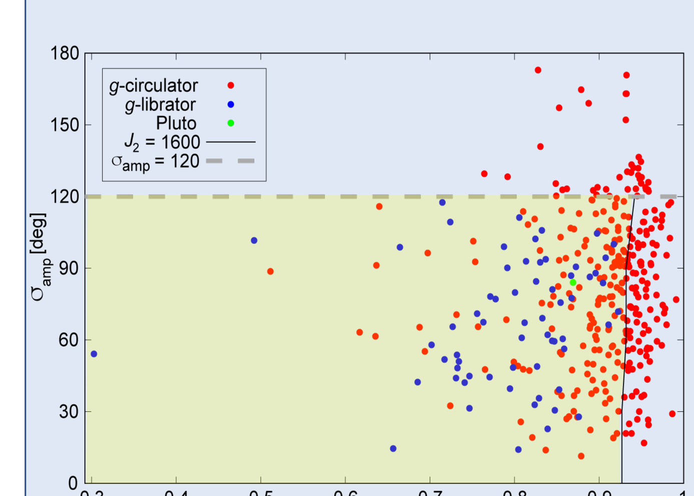
We repeat the numerical quadrature for the Sun-Neptune-Pluto system to investigate the dependence on $\sigma_{\rm amp}$ and J_2. Note that $J_2 = 1600$ is the solar system proxy in the right panel.

When Sun's J_2 is large, the disturbing potential it creates strongly promotes *g*'s precession (i.e. large d*g*/dt), preventing *g* from showing libration
Appearance of stationary points (where d*g*/dt = 0) becomes rare



Dependence of $k^2_{\rm crit}$ on $\sigma_{\rm amp}$ and J_2.
No *g*-libration was found for $\sigma_{\rm amp} > 120°$.

Can our model explain the secular behavior of argument of perihelion of observed Plutinos? – Yes



Relationship between k^2 and $\sigma_{\rm amp}$ of the 363 Plutinos obtained from the direct numerical orbit propagation under the influence of the four giant planets. The red points ● denote *g*-circulators, the blue points ● denote *g*-librators, and Pluto is denoted as the green point ●.

Observed *g*-librating Plutinos ● are confined in the yellow-hatched region on the (k^2, $\sigma_{\rm amp}$) plane
- No *g* librator has $\sigma_{\rm amp}$ larger than 120°
- No *g* librator exists beyond the $J_2 = 1600$ curve

Distribution of the *g* librators can be well explained by the model using
- Sun's J_2 as the proxy of Jupiter, Saturn, Uranus
- Averaged disturbing function with sinusoidal σ

✓ Consideration of additional effects such as below is unnecessary for this purpose
- Neptune's eccentricity ($e_{\rm N} \sim 0.01$)
- 2:1 mean motion "near resonance" between Uranus and Neptune

See also: companion poster #2311 in Poster Session Centaurs, TNOs and Trojans

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